Chapter 5 Third Edition

Basic Twelve-Tone Operations

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Introduction to Post-Tonal theory (Upper Saddle River, NJ: Prentice Hall, 2005)

Twelve-Tone Series

Until now, we have discussed music largely in terms of *unordered* sets of pitch classes. In what follows, we will concentrate on *ordered* sets, which we will call *series*. A series is a line, not a set, of pitch classes. A pitch-class set retains its identity no matter how its pitch classes are ordered. In a series, however, the pitch classes occur in a particular order; the identity of the series changes if the order changes.

A series can be any length, but by far the most common is a series consisting of all twelve pitch classes. A series of twelve different pitch classes is sometimes called a *set* (a usage we will avoid because of the possibility of confusion with unordered pitch-class sets) or *row*. Music that uses such a series as its basic, referential structure is known as *twelve-tone music*.

A twelve-tone series plays many musical roles in twelve-tone music. In some ways it is like a theme, a recognizable "tune" that recurs in various ways throughout a piece. In some ways it is like a scale, the basic referential collection from which harmonies and melodies are drawn. In some ways it is a repository of motives, a large design within which are embedded numerous smaller designs. But it plays a more fundamental role in twelve-tone music than theme, scale, or motive play in tonal music. In tonal music, the scales and even to some extent the themes and motives are part of the common property of the prevailing musical style. From piece to piece and from composer to composer, a great deal of musical material is shared. Tonal music is relatively communal. In twelve-tone music, however, relatively little is shared from piece to piece or composer to composer; virtually no two pieces use the same series. Twelve-tone music is thus relatively contextual. The series is the source of structural relations in a twelve-tone piece: from the immediate surface to the deepest structural level, the series shapes the music.

Basic Operations

Like unordered pitch-class sets, twelve-tone series can be subjected to various operations like transposition and inversion for the sake of development, contrast, and continuity. There is an important basic difference, however. When a set of fewer than twelve elements is transposed or inverted, the content of the set usually changes. When any member of 4–1 (0123), for example, is transposed up two semitones, two new pitch classes will be introduced. The operation of transposition thus changes the content of the collection. When a twelve-tone series is transposed, however, the content remains the same. If you transpose the twelve pitch classes, you just get the same twelve pitch classes, but in a different order. The same is true of inversion. In the twelve-tone system, the basic operations—transposition and inversion—affect order, not content.

The series is traditionally used in four different orderings: prime, retrograde, inversion, and retrograde-inversion. Some statement of the series, usually the very first one in the piece, is designated the prime ordering and the rest are calculated in relation to it. Schoenberg's String Quartet No. 4, for example, begins as shown in Example 5–1.

Example 5–1 Presenting the series—the initial statement is designated P_2 (Schoenberg, String Quartet No. 4).

The melody in the first violin presents all twelve pitch classes in a clear, forthright way. We will consider this the prime ordering for the piece. It begins on D (pitch-class 2), so we will label it P_2.

Figure 5–1 shows P_2 for Schoenberg's String Quartet No. 4 and the interval succession it describes.

ordered pitch-class intervals:	11	8	1	7	10	1	8	8	11	11	5	
P_2	D	C♯	A	B♭	F	E♭	E	C	A♭	G	F♯	B

Figure 5–1

Let's see what happens if we transpose P_2 up seven semitones (see Figure 5–2).

ordered pitch-class intervals:	11	8	1	7	10	1	8	8	11	11	5	
P_9	A	G♯	E	F	C	B♭	B	G	E♭	D	C♯	F♯

Figure 5–2

The order of the pitch classes changes: D was first, now it is toward the end; A was third, now it is first; and so on. In fact, no pitch class occupies the same order position it did. The content, of course, is the same (both P_2 and P_9 contain all twelve pitch classes) and, more important, so is the interval succession. That particular interval succession is what defines the prime ordering of this series. We can produce that succession beginning on any of the twelve pitch classes. P_0 is the prime ordering beginning with pitch class 0; P_1 is the prime ordering beginning with pitch class 1; and so on. There are twelve different forms of the prime ordering: $P_0, P_1, P_2, \ldots, P_{11}$.

As for the other orderings (retrograde, inversion, and retrograde-inversion), we can think of them either in terms of their effect on the pitch classes or their effect on the intervals. In terms of pitch classes, the retrograde ordering simply reverses the prime ordering. What happens to the interval succession when P_2 is played backwards (an ordering called R_2)? Figure 5–3 demonstrates.

ordered pitch-class intervals:	7	1	1	4	4	11	2	5	11	4	1	
R_2	B	F♯	G	A♭	C	E	E♭	F	B♭	A	C♯	D

Figure 5–3

The interval succession is reversed and each interval is replaced by its complement mod 12 (1 becomes 11, 2 becomes 10, etc.). As with the prime ordering, there are twelve different forms of the retrograde ordering: $R_0, R_1, R_2, \ldots, R_{11}$. (Remember that R_0 is the retrograde of P_0, R_1 the retrograde of P_1, and so on. R_0 thus ends rather than begins on 0.)

The inversion of the series involves inverting each pitch class in the series: pitch class 0 inverts to 0, 1 inverts to 11, 2 inverts to 10, 3 inverts to 9, and so on.

Figure 5–4 shows the interval succession for I_7, the inverted ordering that begins on pitch class 7.

ordered pitch-class intervals:		1		4		11		5		2		11		4		4		1		1		7	
I_7	G		A♭		C		B		E		F♯		F		A		C♯		D		E♭		B♭

Figure 5–4

The interval succession here is the same as that of the prime ordering, but each interval is replaced by its complement mod 12. The intervals are the same as in the retrograde, but in reverse order. As with the prime and retrograde orderings, we can reproduce this interval succession beginning on any of the twelve pitch classes. The twelve resulting series-forms will be called I_0, I_1, I_2, . . . , I_{11}.

The retrograde-inversion of the series is simply the retrograde of the inversion. Figure 5–5 shows the interval succession for RI_7 (I_7 played backwards).

ordered pitch-class intervals:		5		11		11		8		8		1		10		7		1		8		11	
RI_7	B♭		E♭		D		C♯		A		F		F♯		E		B		C		A♭		G

Figure 5–5

The interval succession here is similar to that of the other three transformations. It is particularly interesting to compare it to that of the prime ordering. In terms of pitch classes, the two orderings seem far apart: each is the upside-down-and-backwards version of the other. In terms of intervals, however, the two are quite similar: they have the same intervals in reverse order. Compared to the retrograde, the retrograde-inversion has the complementary intervals in the same order; compared to the inversion, it has the complementary intervals backwards. As with the other three transformations, the retrograde-inversion can begin on any of the twelve pitch classes. The resulting series-forms are named RI_0 (the retrograde of I_0), RI_1 (the retrograde of I_1), . . . , RI_{11} (the retrograde of I_{11}).

For any series, we thus have a family of forty-eight series-forms: twelve primes, twelve retrogrades, twelve inversions, and twelve retrograde-inversions. All the members of the family are closely related in terms of both pitch classes and intervals. Figure 5–6 shows the intervals described by the four different orderings of the series from Schoenberg's Quartet No. 4.

	ordered pitch-class intervals										
Prime:	11	8	1	7	10	1	8	8	11	11	5
Retrograde:	7	1	1	4	4	11	2	5	11	4	1
Inversion:	1	4	11	5	2	11	4	4	1	1	7
Retrograde-Inversion:	5	11	11	8	8	1	10	7	1	8	11

Figure 5–6

Notice the predominance of intervals 1, 4, 8, and 11 and the complete exclusion of 3 and 9 in all four orderings (and thus in all forty-eight series-forms). Because of these shared intervallic features (and many other features to be discussed later), the forms of a series are closely related to one another. Each of them can impart to a piece the same distinctive sound.

Figure 5–7 summarizes the intervallic relationships among series forms.

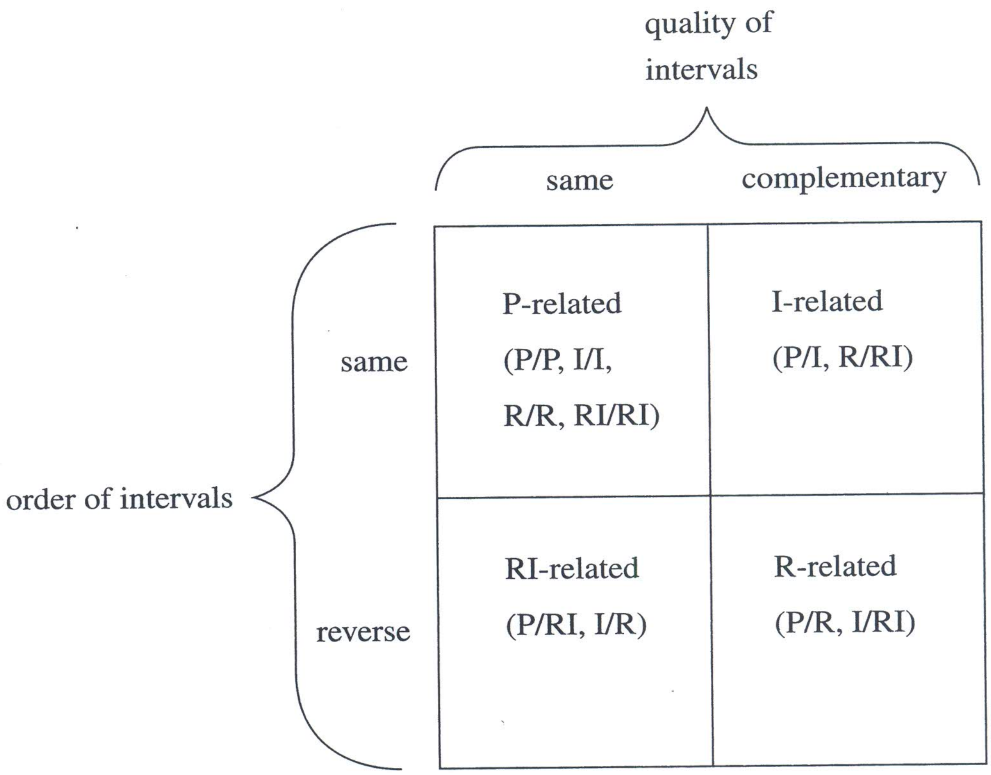


Figure 5–7

Series that have the same ordering (P and P, I and I, R and R, or RI and RI) are said to be prime-related and have the same intervals in the same order. Series that are related to each other by inversion (P and I, R and RI) have the complementary intervals in the same order. Series that are retrograde-inversionally-related (P and RI, I and R) have the same intervals in reverse order. Series that are retrograde-related (P and R, I and RI) have the complementary intervals in reverse order.

In studying a twelve-tone piece, it is convenient to have at hand a list of all forty-eight forms of the series. We could just write out all forty-eight either on staff paper or using the pitch-class integers. More simply, we could write out the twelve primes and the twelve inversions (using the musical staff, letter names, or pitch-class integers) and simply find the retrogrades and retrograde-inversions by reading backwards. The simplest way of all, however, is to construct what is known as a "12 × 12 matrix." To construct such a matrix, begin by writing P_0 horizontally across the top and I_0 vertically down the left side (see Figure 5–8).

0	11	7	8	3	1	2	10	6	5	4	9
1											
5											
4											
9											
11											
10											
2											
6											
7											
8											
3											

Figure 5–8

Then write in the remaining prime orderings in the rows from left to right, beginning on whatever pitch class is in the first column. The second row will contain P_1, the third row will contain P_5, and so on (see Figure 5–9).

I ↓

P →   ← R

0	11	7	8	3	1	2	10	6	5	4	9
1	0	8	9	4	2	3	11	7	6	5	10
5	4	0	1	8	6	7	3	11	10	9	2
4	3	11	0	7	5	6	2	10	9	8	1
9	8	4	5	0	10	11	7	3	2	1	6
11	10	6	7	2	0	1	9	5	4	3	8
10	9	5	6	1	11	0	8	4	3	2	7
2	1	9	10	5	3	4	0	8	7	6	11
6	5	1	2	9	7	8	4	0	11	10	3
7	6	2	3	10	8	9	5	1	0	11	4
8	7	3	4	11	9	10	6	2	1	0	5
3	2	10	11	6	4	5	1	9	8	7	0

↑ RI

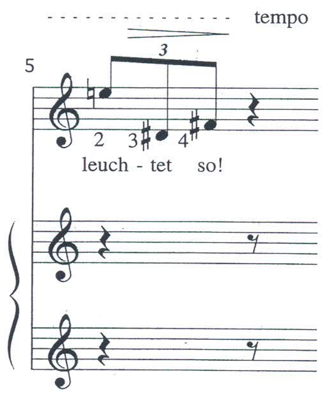
Figure 5–9

The same matrix also could be written using letter names instead of pitch-class integers. The rows of the matrix, reading from left to right, contain all of the prime forms and, reading from right to left, the retrograde forms. The columns of the matrix reading from top to bottom contain all of the inverted forms and, from bottom to top, the retrograde-inversion forms.

The matrix thus contains an entire small, coherent family of forty-eight closely related series-forms: twelve primes, twelve retrogrades, twelve inversions, and twelve retrograde-inversions. All of the essential pitch material in a twelve-tone piece is normally drawn from among those forty-eight forms. In fact, most twelve-tone pieces use far fewer than forty-eight different forms. The material thus is narrowly circumscribed yet permits many different kinds of development. A composer builds into the original series (and thus into the entire family of forty-eight forms) certain kinds of structures and relationships. A composition based on that series can express those structures and relationships in many different ways.

One good way of getting oriented in a twelve-tone work is by identifying the series-forms. This is informally known as "twelve-counting," and it can provide a kind of low-level map of a composition. The first step in twelve-counting is to identify the series. It is usually presented in some explicit way right at the beginning of the piece, but sometimes a bit of detective work is needed. For an example, let's turn back to Webern's song "Wie bin ich froh!" discussed in Analysis 1. The melody for the passage we discussed, measures 1–5, presents the twelve-tone series for the piece, and then repeats its first four notes (see Example 5–2).

(continued)



Example 5–2 The melody presents the P_7 form of a twelve-tone series (Webern, "Wie bin ich froh!").

We will designate that form of the series as P because it is so prominent and easy to follow. Notice the usual twelve-counting procedure of identifying the order-position each pitch occupies in the series-form (G is first, E is second, and so on).

Now the problem is to identify the series-forms used in the accompaniment. We could construct a 12 × 12 matrix. Then we could take the first few notes in the accompaniment (F♯, F, D) and see which of the forty-eight forms begins like that. That would work perfectly well, but instead let's try a different, interval-oriented approach. Look at the succession of ordered pitch-class intervals described by P_7 (see Figure 5–10).

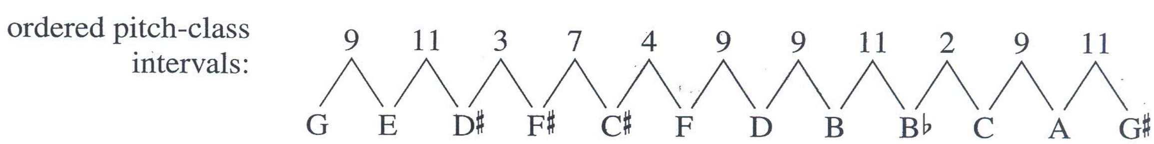


Figure 5–10

Now look at the ordered pitch-class intervals described by the first five notes of the accompaniment (see Figure 5–11).

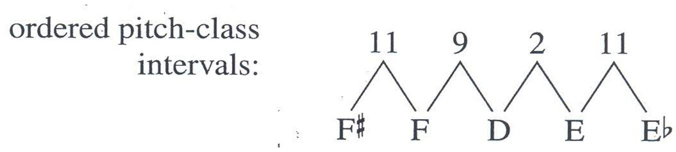


Figure 5–11

It starts out with the same intervals that P_7 ended with, but in reverse order. That means we are dealing with an RI-form. At what transposition level? Just add the first note in the accompaniment (F♯) to the last note in P_7 (G♯), the second note in the accompaniment (F) to the second-to-last note in P_7 (A), and so on. In this way, we calculate the index number that maps these series-forms onto one another. The sum in each case is 2. So the accompaniment begins with RI_7, because $7 + 7 = 2$. Each note in RI_7, added to the corresponding note in P_7, sums to the index number 2.

Webern's two series forms, P_7 and RI_7, are related to each other at T_2I, and there is a strong sense in the music of balancing around the prescribed axis C♯–G. The G in particular plays a role as a center of inversional symmetry.

This passage uses only P_7 and RI_7, and the entire song uses only these two forms and their retrogrades (see Example 5–3).

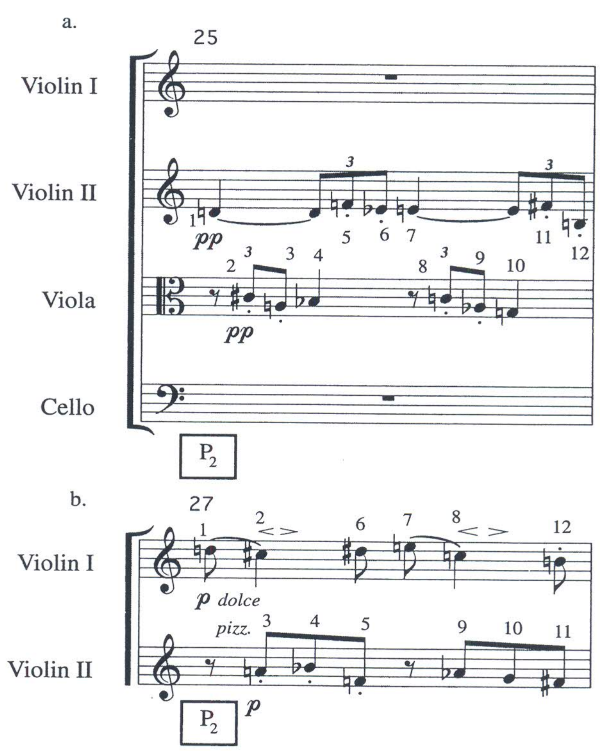
Example 5–3 A "twelve-count" of melody and accompaniment.

Notice that occasionally a single note can be simultaneously the last note of one series-form and the first note of the next. The G in the accompaniment in measure 2, for example, is both the last note in RI_7 and the first note in P_7. This kind of overlap is typical of Webern. A twelve-count like this doesn't do much to help us hear the song better—the intervallic relationships discussed in Analysis 1 are probably more useful in that way—but it does give a rough structural outline of the piece. It also gives a clarifying context for those intervallic relationships.

There is nothing mechanical about either the construction of the series or its musical development in a composition. A composer of tonal music is given certain materials to work with, including, most obviously, diatonic scales and major and minor triads. The composer of twelve-tone music must construct his or her own basic materials, embedding them within the series. When it comes time to use those basic materials in a piece of music, a twelve-tone composer, like a tonal composer, does so in the way that seems musically and expressively most congenial. A good composer doesn't just lay series-forms end-to-end any more than Mozart simply strings scales together.

Once a series has been constructed, a process we will describe more later, just think how many compositional decisions are still required to turn it into music. Should the notes be sounded one at a time or should some of them be heard simultaneously? In what registers should they occur? Played by what instruments? With what durations? What articulations? It is like being given a C-major scale and told to compose some music. There are certain restrictions, but a great deal of freedom as well.

Example 5–1 showed the beginning of Schoenberg's String Quartet No. 4, where P_2 is presented as a singing melody in the first violin. Example 5–4 shows two other statements of P_2, both from the opening section of the piece.



Example 5–4 Two additional statements of P_2 (Schoenberg, String Quartet No. 4).

The musical idea is recognizable in each case, but wonderfully varied. Schoenberg takes a basic shape, then endlessly reshapes it. The construction of the series, the choice of series-forms, and, most important, the presentation of the series, are musical decisions based on hearable musical relationships.

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