The Twentieth-Century Canon: An Analysis of Luigi Dallapiccola's Canonic Works from his *Quaderno Musicale di Annalibera*

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Thesis submitted to the
Faculty of Graduate and Postdoctoral Studies
In partial fulfillment of the requirements
For the MA in Music Theory Program

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TABLE OF CONTENTS

LIST OF MUSICAL EXAMPLES

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

The compositional technique of cross partitioning is one of Luigi Dallapiccola's most used twelve-tone devices. Through a detailed analysis of three contrapuntal canonic movements from Dallapiccola's *Quaderno Musicale di Annalibera*, I examine his use of cross partitioning as a motivic tool and as a referential collection. The development of the BACH motive and the derivation of tone-row statements reflects on Dallapiccola's extensive use of cross partitioning and his compositional principles used to achieve a sense of *polarity*. Upon a preliminary analysis based on set-theory analysis set out by Joseph Straus I draw an interpretive analysis through Alegant's cross partitioning model as well as develop my own set of parameters for interpretation in regards to *polarity* which is based on intervallic stability.

Keywords: Luigi Dallapiccola, *Quaderno Musicale di Annalibera*, twelve-tone, polarity, centricity, cross partition, Brian Alegant, set-theory, Joseph Straus, canon, BACH motive

ACKNOWLEDGEMENTS

Without Murray Dineen's endless encouragement and support this thesis would not have been possible. His ability to create a comfortable and calm learning environment does not go unnoticed or under appreciated. I would also like to thank my fellow graduate students for their invaluable advice and moral support. Lastly, special thanks to my good friend Sinae Kim for her cooperation during our late night brain storming sessions.

CHAPTER ONE

INTRODUCTION

Luigi Dallapiccola’s *Quaderno Musicale di Annalibera*, a collection of eleven short pieces for piano solo, was written for the Pittsburgh International Contemporary Music Festival of 1952 and was dedicated to his daughter on her eighth birthday. The *Quaderno* contains pieces that range from 27 seconds to two minutes and 45 seconds in length. As a tribute to J.S. Bach, it includes the BACH motive throughout, and alludes to J.S. Bach through its title, recalling the *Notebook for Anna Magdalena*. Bach's musical notebook, likewise, contains a small sub collection of contrapuntal pieces in the canonic style. Each one of the contrapuntal works in the Dallapiccola collection is extrapolated from a single twelve-tone series, which the entire *Quaderno* is based on. The three canonic movements studied here are all different in kind: *Contrapunctus Primus* is a mensural canon which becomes a perpetual double mensuration canon by inversion, *Contrapunctus Secundus* is a canon by inversion, and *Contrapunctus Tertius* is a crab canon. The purpose of this study is to provide a detailed analysis of Dallapiccola's three canonic movements and to provide justification for his compositional choices based on recent studies by Brian Alegant and Joseph Straus in combination with Dallapiccola's principles of composition. Dallapiccola's extensive use of cross partitioning is thoroughly examined in order to achieve the compositional goal of polarity. This study begins with a brief overview of the available literature written on Dallapiccola and his works along with a description of the literatures relevance to this analysis. The literature comprises mostly of journal articles, dissertations, and recently published books.

LITERATURE REVIEW

Scholarly literature written on Dallapiccola comprises mostly biography, lyrical tributes, and analysis of his vocal works. Much is known about his instrumental works and his contrapuntal writing, but his twelve-tone technique needs to be explored in greater detail. Many of his works have yet to be thoroughly discussed in a scholarly environment. This study provides such an environment with new suggestions of Dallapiccola's compositional techniques as well as introduces new research ideas to be used to initiate further analytical studies on his works. The literature that exists on the analysis of Dallapiccola's works is mostly in English and comprises mainly dissertations and journal articles, with the exception of two recently published books by Brian Alegant and Raymond Fearn. What follows is a brief description of the literature most relevant to this study.

Brian Alegant, *The Twelve-Tone Music of Luigi Dallapiccola (2010)*

Brian Alegant's recent book *The Twelve-Tone Music of Luigi Dallapiccola (2010)*, is an analytical approach to Dallapiccola's lesser studied compositions. Alegant focuses on *how* Dallapiccola composed with twelve-tones, rather than *why.* Essentially, by focusing on compositions which have not made an appearance in the Dallapiccola critical literature, Alegant comes to a fuller understanding of Dallapiccola's technique. Alegant achieves this by examining fewer works in greater depth and implementing recent post-tonal concepts by Allen Forte, David Lewin, Andrew Mead, Robert Morris, Joseph Straus, and of his own creation. The book is organized as a timeline stretched over a thirty year period, which is broken down into four compositional phases. Alegant categorizes Dallapiccola's musical ideas as either Schoenbergian or Weberian in character in attempting to demonstrate how Dallapiccola came about a particular technique. This book provides this study with the preliminary analytical model used to analyse Dallapiccola's three movements. Alegant's analytical models are used to identify and contextualize Dallapiccola's use of cross partitioning in his canonic works.

Brian Alegant, *Cross-Partitions as Harmony and Voice Leading in Twelve-Tone Music (2001)*

Brian Alegant's article *Cross-Partitions as Harmony and Voice Leading in Twelve-Tone Music (2001)* from *Music Theory Spectrum* is one of the most exhaustive writing on the principle of cross partitioning. Though the topic was briefly introduced by Donald Martino, Alegant is the first to explain the concept thoroughly by means of specific examples from the works of Webern, Schoenberg, and Dallapiccola. Alegant's article shows how each composer has his own signature use of the twelve-tone device. The section on Dallapiccola's use of cross partitioning helps this study illustrate Dallapiccola's use of the technique and offers insight to his creative ways of using cross partitioning.

Luigi Dallapiccola, *On the Twelve-Note Road (1951)*

Luigi Dallapiccola's essay *On the Twelve-Note Road (1951)* from *Music Survey* is a chronicle of his arrival at the twelve-tone system and his conception of the principles he employs in his compositions. He reflects on the inadequacy of the tonal system and his reasoning for taking on the twelve-tone system – to achieve optimum expressivity. He writes on musical concepts such as arrangement, canon, polarity, and time. Though he did not coin the term, Dallapiccola writes about cross partitioning – "Before reaching the rhythmic and melodic definition of the series, we may find it compressed into a single chord of twelve-notes, two chords of six notes, three of four, four of three notes, or even six two-note chords…"[1] Dallapiccola is suggesting that an arrangement of any of the listed configurations can be used as a referential guide for the listener. Depending on the arrangement of the notes, certain interval classes will proliferate out of the arrangement. These interval classes are what drive the composition, serving as a centric music idea, which Dallapiccola refers to as *polarity*. Perhaps one of the most valuable pieces of literature, this essay provides this study with Dallapiccola's views and understandings of the twelve-tone system and how he came about them.

David L. Mancini, *Twelve-Tone Polarity in Late Works of Luigi Dallapiccola (1986)*

David L. Mancini's *Twelve-Tone Polarity in Late Works of Luigi Dallapiccola (1986)* from the *Journal of Music Theory* is an analytical study of three excerpts which illustrate the organization of Dallapiccola's twelve-tone method in achieving a quality called "polarity". His analysis suggests, and does so quite successfully, that polarity can be established through one centric musical aspect – a pitch-class set. He introduces the concept of polarity as Dallapiccola defined it in his essay *On the Twelve-Note Road (1951)*, but adds that a piece of music could contain more than one centric aspect and could move to another. Mancini also proposes that the centric aspect of a piece (i.e. "tonic") does not have to be a single pitch, like in tonal music, but could indeed be an interval class or a pitch-class set. Mancini's article raises questions in regards to Dallapiccola's methods of achieving attractive forces, but not achieving a sense of tonic. He further questions Dallapiccola's criteria in defining what constitutes an attractive force

[1] Luigi Dallapiccola, "On the Twelve-Tone Road", *Music Survey 4* (1951): 329.

and their existence in the music. Mancini's questions help this study realize the flexibility of Dallapiccola's understand of polarity.

Hans Nathan, *Luigi Dallapiccola: Fragments from Conversations (1966)*

Hans Nathan's *Luigi Dallapiccola: Fragments from Conversations (1966)* from *The Music Review* is a conversational dialogue between Nathan and Dallapiccola on various musical concepts such as dodecaphony, atonality and tonality, neo-classicism, harmony, and counterpoint. Nathan questions Dallapiccola on his tone-row construction and his conception of a tone-row. This information allows this study to focus on the musical aspects that were most important to Dallapiccola. Dallapiccola speaks explicitly on the compositional process of the *Canti di Liberazione*, a valuable insight for this study as Dallapiccola uses the same twelve-tone series for the *Quaderno*.

John Macivor Perkins, *Dallapiccola's Art of Canon (1963)*

John Macivor Perkins article *Dallapiccola's Art of Canon (1963)* from *Perspectives of New Music* focuses primarily on the rhythmic structure and rhythmic complexities of Dallapiccola's canon. Perkins touches on Dallapiccola's transformations of the mensuration canon, the crab canon, and metrical accenting, which Perkins refers to as *floating*. Avoiding any analyses, this article better serves as a presentation of Dallapiccola's principles of writing. This article gives this study and outline of Dallapiccola's canonic structures. In particular, rhythmic structures and metrical contexts are discussed which serve as guidelines in defining the rhythmic structures of Dallapiccola's canons.

Thomas DeLio, *A Proliferation of Canons: Luigi Dallapiccola's "Goethe Lieder No. 2" (1985)*

Thomas DeLio's journal article *A Proliferation of Canons: Luigi Dallapiccola's "Goethe Lieder No. 2" (1985)* from *Perspectives of New Music* discusses on Dallapiccola's ability to work with a series of complex relationships and construct them into a signature canonic framework. DeLio touches on hexachordal and trichordal partitionings and the means by which Dallapiccola effects them. A similar type of partitioning is found throughout the contrapuntal works of the *Quaderno*. Although the *Goethe Lieder* was written after the *Quaderno*, both were written in the same compositional period, according to Alegant. Compositional techniques used in the *Goethe Lieder* that are also used in the *Quaderno* can further reinforce the findings of this study. One may be able to trace the development of a particular compositional technique through a chronological series of compositions, but this lies beyond the scope of this thesis.

Raymond Fearn, *The Music of Luigi Dallapiccola (2003)*

Raymond Fearn's book *The Music of Luigi Dallapiccola (2003)* is an analytical survey covering all of Dallapiccola's works like Alegant's book, Fearn breaks down Dallapiccola's compositional career into chronological periods. Fearn's, however, analysis rarely goes beyond basic observations. In his chapter containing the movements from *Quaderno Musicale di Annalibera*, Fearn presents only tone-row identifications and speaks briefly on the BACH motive imbedded in the first movement. This book brings the awareness of Dallapiccola's compositions to higher level without becoming too technical.

Rosemary Brown, *Continuity and Reoccurrence in the Creative Development of Luigi Dallapiccola (1977)*

Rosemary Brown's dissertation *Continuity and Reoccurrence in the Creative Development of Luigi Dallapiccola (1977)* is widely considered as one of the most knowledgeable resources on Dallapiccola. Brown tackles such subjects as stylistic development, symmetry and parallelism, symbolism, pedal effects, cyclic techniques, contrapuntal forms and techniques, and rhythmical and metrical experimentation. Brown, however, manages to present only the most prominent examples from Dallapiccola's work and avoids going into detail with an individual piece.

Hans Nathan, *The Twelve-Tone Compositions of Luigi Dallapiccola (1958)*

Hans Nathan's article *The Twelve-Tone Compositions of Luigi Dallapiccola (1958)* from *Musical Quarterly* is one of the first published writings to discuss Dallapiccola's concept of the twelve-tone system. Through a brief overview of several of Dallapiccola's compositions, Nathan discusses: symmetry, chromaticism, expressionism, rhythmic structure, and tone-row arrangement, and makes comparisons to some of the great composers of the twentieth century – Bartok, Berg, Schoenberg, Wagner, and Webern. At the time of its publication, Nathan's article considered Dallapiccola to be in the most significant phase of his compositional development. Nathan's article also includes an index of all of Dallapiccola's twelve-tone compositions.

CHAPTER TWO

METHODOLOGY

This chapter outlines and explains the primary methodology used in this study as applied to three contrapuntal movements from Luigi Dallapiccola's *Quaderno Musicale di Annalibera*. Primarily, through the application of set-theory analysis set out by Joseph Straus, I am able to explore tone-row relationships and the concept of centricity in post-tonal music. From this, I am able to draw upon recent theories on cross partitioning developed by Brian Alegant to interpret Dallapiccola's use of the compositional technique, both as a motivic tool and a vehicle of delivery for centricity. Both theoretical frameworks contribute to Dallapiccola's understanding of *polarity*. Through the construction of matrices, invariant relationships aqre extracted to show Dallapiccola's intricate method of tone-row construction. Invariant relationships include both discrete and non-discrete chords, as well as non-segmental invariance. It should be noted that Dallapiccola was never taught by the Viennese masters or their pupils. He adopted the twelve-tone system before the publications of René Leibowitz's books and had to rely entirely on his own abilities based on the scores he was able to obtain.

The Twelve-Tone Series – Preliminaries

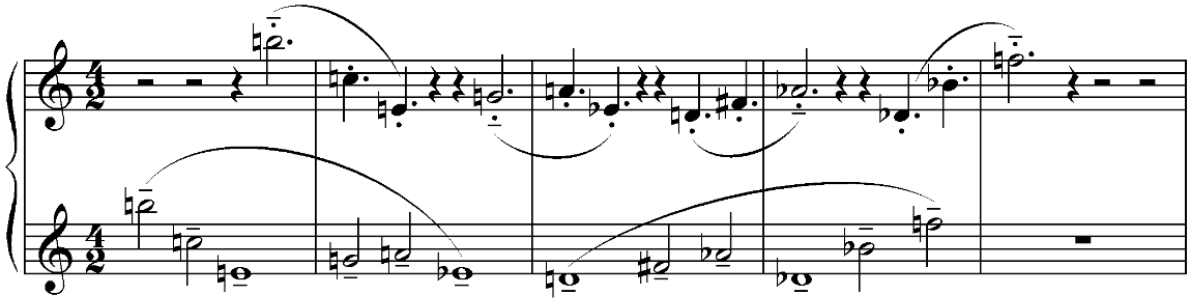
I begin my analysis based on Straus's basic theoretical concepts of post-tonal music taken from his *Introduction to Post-Tonal Theory (2005)*. A twelve-tone series is an arrangement of all twelve pitch classes, which occur in a particular order and is used as a basic musical structure for a composition.[2] A series also could be a basic referential collection of pitch classes which could include thematic material, scales, melodies, or any other type of musical role employed in a composition. Once a twelve-tone series is defined, one can subject it to transformation such as inversion, retrograde, and retrograde-inversion to generate a family of forty-eight tone row forms. Each of these forty-eight tone row forms is related to the original prime form and to one another.

I have adopted the “12 x 12 matrix" design, commonly referred to as the magic square, as it is the most convenient tool to have on hand when analyzing a twelve-tone work. Figure 2.2 is Dallapiccola’s matrix; which he uses as a referential tool throughout the entire *Quaderno*. The matrix will contain all related forms of the series and is essential in discovering structural relationships among the tone-rows. To derive the matrix, one must extract an ordering of the pitches from the music, normally presented at the onset of the piece.

Contrapunctus Primus presents the twelve-tone series at the beginning of the piece in a linear fashion. Example 2.1 shows the ordering of all twelve pitch classes taken from the opening measures of *Primus*. Included below the score is the ordering of the tone-row and the registral placement of each pitch.

[2] Straus, Joseph. *Introduction to Post-Tonal Theory* (New Jersey: Prentice Hall, 2005), 182.

Example 2.1 - Measures 1-5 of *Primus*.



Order: *0 1 2 3 4 5 6 7 8 9 t e*

P_e: e 0 4 7 9 3 2 6 8 1 t 6

With an ordering of the series, one can format it so it conforms to a "12 x 12 matrix". The matrix gives the analyst a concise presentation of all forty-eight tone-rows in all four forms. Having the matrix on hand makes identifying tone-rows in the piece much easier and also allows the analyst to visualize the type of relationship between tone-rows. To construct the matrix, one must first transpose the series so that the first note is 0, by using *mod 12*. It is necessary to use a transpositional operation, T_n, where *n* is the number of semitones by which each pitch class is increased. For this series, transpositional operation T_1 is required to find the prime form that begins with 0, (see figure 2.1).

Figure 2.1 – Transposition of P_e through operation T_1 resulting in P_0.

e 0 4 7 9 3 2 6 8 1 t 5
\+ 1 1 1 1 1 1 1 1 1 1 1 1
0 1 5 8 t 4 3 7 9 2 e 6

To construct the matrix, the prime form of the series (P_0) will be viewed horizontally across the top, and its inversion will appear vertically down the left side. The inversion is found by determining each of the pitch classes' complement using *mod 12*. Figure 2.2 shows a complete matrix derived from the ordering of pitch classes found in *Primus*.

Figure 2.2 – The matrix of Dallapiccola's twelve-tone series.

Inversion

Prime

0	1	5	8	t	4	3	7	9	2	e	6
e	0	4	7	9	3	2	6	8	1	t	5
7	8	0	3	5	e	t	2	4	9	6	1
4	5	9	0	2	8	7	e	1	6	3	t
2	3	7	t	0	6	5	9	e	4	1	8
8	9	1	4	6	0	e	3	5	t	7	2
9	t	2	5	7	1	0	4	6	e	8	3
5	6	t	1	3	9	8	0	2	7	4	e
3	4	8	e	1	7	6	t	0	5	2	9
t	e	3	6	8	2	1	5	7	0	9	4
1	2	6	9	e	5	4	8	t	3	0	7
6	7	e	2	4	t	9	1	3	8	5	0

Retrograde

Retrograde-Inversion

Once the matrix is created, an analyst now has a referential collection of pitch classes used to identify tone rows in their relationships within a work.

A series can be further partitioned into smaller segments of two notes, three notes, four notes, and six notes – dyads, trichords, tetrachords, and hexachords. Each part is considered to be a subset of a series that shapes the intervallic characteristics of the structure. Due to the dyadic, trichordal, and tetrachordal structure in Dallapiccola's three contrapuntal movements, close attention will be paid to these structures in the series. Figure 2.3 shows the matrix and outlines its discrete subset structure as well as the names of set classes according to Allen Forte's *The Structure of Atonal Music (1973)*. These subsets are considered to be discrete because they are not overlapping with one another and can be evenly distributed over the total series or tone-rows using all pitch classes.

Figure 2.3 – Discrete chords of Dallapiccola's series.

		3-4			3-8			3-8			3-11	
	0	1	5	8	t	4	3	7	9	2	e	6
	e	0	4	7	9	3	2	6	8	1	t	5
4-20	7	8	0	3	5	e	t	2	4	9	6	1
	4	5	9	0	2	8	7	e	1	6	3	t
	2	3	7	t	0	6	5	9	e	4	1	8
	8	9	1	4	6	0	e	3	5	t	7	2
4-18	9	t	2	5	7	1	0	4	6	e	8	3
	5	6	t	1	3	9	8	0	2	7	4	e
	3	4	8	e	1	7	6	t	0	5	2	9
	t	e	3	6	8	2	1	5	7	0	9	4
4-26	1	2	6	9	e	5	4	8	t	3	0	7
	6	7	e	2	4	t	9	1	3	8	5	0
				6-31						6-31		

These sub-set collections of discrete chords are used to guide the listener through a composition via a chain of invariance. Elements of the series which are preserved under any type of transformation are considered to be invariant. These invariants are often employed as trichords, tetrachords, and hexachords and are then used to guide the listener along the composition (as an entire collection in any particular order of the twelve pitch classes becomes difficult to aurally retain).

Commonly, a composer will use invariant elements to associate one musical idea with another that is subject to some type of transformation. An *invariant* is, “any musical quality or relationship preserved when the series is transformed.”[3] In order to create these types of relationships, one must construct a twelve-tone series through a carefully planned arrangement of the pitches.

General Properties of the Series

In the material which follows, I have catalogued the chordal invariances in the series Dallapiccola used for the entire collection of the *Quaderno*. By cataloging chordal invariances the analyst is able to simply refer back to the tables to assist in identifying tone-row transformations within a piece of music. Although Dallapiccola does not use all of the invariant relationships in the pieces that are involved in this study, it is worth considering them all because the series is used as a referential collection of pitch classes throughout. Creating invariance in a series can, at times, be a simple task. However, the more invariants involved in the transformation and the type of operation used to map the relationship onto itself will make creating invariance a challenging task. Included in this catalogue are the types of invariance and the operations required to achieve them. I have included several matrices used to highlight the invariants, as well as tables that show the operations and transformations each tone row must undergo to achieve invariance. Also included is an ordering repositioning pattern to show the

[3] Straus, Joseph. *Introduction to* (New Jersey: Prentice Hall, 2005), 195.

movement of the pitch classes through the transformation. Dallapiccola strongly believed that the rearrangement and reordering of series could change its identification and allow it to take on new meaning. Lastly, I have coined new terms to describe the various types of invariances – *separable invariance* and *double order invariance*. These terms will be defined as they are introduced.

Hexachordal Combinatoriality

Dallapiccola constructs the series in such a way that is it hexachordally combinatorial by inversion. The first hexachord of any given tone-row will map onto itself as the second hexachord of another tone-row under an inversional transformation, creating a hexachordally I-combinatorial series. Figure 2.4 shows that the combination of two tone-rows could allow for the full presentation of the series in an aggregate fashion, as opposed to linearly (each hexachord labeled accordingly as H_1 and H_2). An aggregate presentation does not allow for an official ordering of series until each element is presented in a linear fashion.

Figure 2.4 – Tone-rows P_0 and I_7 are hexachordally I-combinatorial under operation T_7I.

	H_1						H_2					
P_0	0	1	5	8	t	4	3	7	9	2	e	6
I_7	7	6	2	e	9	3	4	0	t	5	8	1
	H_2						H_1					

Aggregate | Aggregate

Each prime form of the series is will paired with a related tone-row through an inversional operation. Table 2.1 shows the operations required to achieve the hexachordally I-combinatorial relationship.

Table 2.1 – A list of operations require for hexachordal combinatoriality.

Tone Rows	Operation
$P_0 - I_7$	T_7I
$P_e - I_6$	T_5I
$P_7 - I_2$	T_9I
$P_4 - I_e$	T_3I
$P_2 - I_9$	T_eI
$P_8 - I_3$	T_eI
$P_9 - I_4$	T_1I
$P_5 - I_0$	T_5I
$P_3 - I_t$	T_1I
$P_t - I_5$	T_3I
$P_1 - I_8$	T_9I
$P_6 - I_1$	T_7I

Each pair of hexchordally I-combinatorial tone-rows is further paired by sharing the same T_nI operation with another set of combinatorial tone-rows. The operations used to achieve the combinatoriality all belong to the same operational family, which I refer to as *odd stream invariance*.[4]

Pentachordal Invariance

Pentachordal invariance is unique in that it cannot partition the twelve-tone row into subsets of the same cardinality. Pentachordal invariance is thus significant in the twelve-tone works. The shaded pentachord in figure 2.5a is invariant at a transpositional operation, T_1. The

[4] *Odd stream operations* – a transformational operation where *n* equals an odd number, T_n or T_nI.

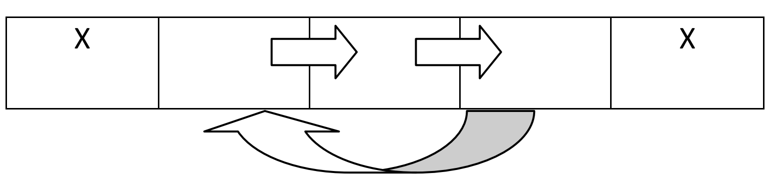
first and last order positions remain in the same position, but the inner positions will move one order position to the right. Note that the pentachord occupies order positions 2 through 6, but under transposition it will occupy order positions 5 through 9. Figure 2.5a highlights the invariant pentachord in the matrix.

Figure 2.5a – Pentachordal invariance.

0	1	5	8	t	4	3	7	9	2	e	6
e	0	4	7	9	3	2	6	8	1	t	5
7	8	0	3	5	e	t	2	4	9	6	1
4	5	9	0	2	8	7	e	1	6	3	t
2	3	7	t	0	6	5	9	e	4	1	8
8	9	1	4	6	0	e	3	5	t	7	2
9	t	2	5	7	1	0	4	6	e	8	3
5	6	t	1	3	9	8	0	2	7	4	e
3	4	8	e	1	7	6	t	0	5	2	9
t	e	3	6	8	2	1	5	7	0	9	4
1	2	6	9	e	5	4	8	t	3	0	7
6	7	e	2	4	t	9	1	3	8	5	0

Figure 2.5b demonstrates how the elements move about within the pentachordal invariance shown through the order repositioning pattern (order positions 01234 becomes 03124).

Figure 2.5b – Order Repositioning Pattern – 01234 → 03124



The non-shaded pentachord is invariant at a transpositional operation; T_e. The relationship of the complementary operations of these two pentachords (T_1 and T_e) is interesting because both pentachords are invariant in the same portion of the tone row and use the same order positioning pattern. Table 2.8 lists the transpositional operations, T_1 and T_e, required to map the same pentachord onto a related tone-row.

Table 2.2 – List of transpositional operations for pentachordal invariance.

Tone Rows	Operation	Tone Rows	Operation
$P_0 – P_1$	T_1	$I_0 – I_e$	T_e
$P_e – P_0$	T_1	$I_1 – I_0$	T_e
$P_7 – P_8$	T_1	$I_5 – I_4$	T_e
$P_4 – P_5$	T_1	$I_8 – I_7$	T_e
$P_2 – P_3$	T_1	$I_t – I_9$	T_e
$P_8 – P_9$	T_1	$I_4 – I_3$	T_e
$P_9 – P_t$	T_1	$I_3 – I_2$	T_e
$P_5 – P_6$	T_1	$I_7 – I_6$	T_e
$P_3 – P_4$	T_1	$I_9 – I_8$	T_e
$P_t – P_e$	T_1	$I_2 – I_1$	T_e
$P_1 – P_0$	T_1	$I_e – I_t$	T_e
$P_6 – P_7$	T_1	$I_6 – I_5$	T_e

Invariances that require T_1 operations will require an equal and opposite operation to return a pentachord back to its original form. All T_1 operations require the use of T_e to return back to its original form and all T_e operations require the use of T_1 to return back to its original form.

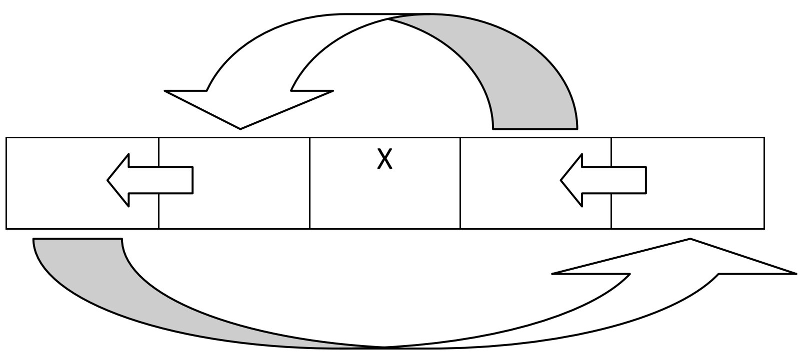
Another type of pentachordal invariance is *separable invariance*[5] – each segment of the tone-row has a counterpart in the related tone-row. Each pentachord shares the same order repositioning pattern and the dyads that separate the pentachords occupy the same space, both as pitch classes and as order positions. Figure 2.6a and b highlight the separable invariance and the order repositioning pattern.

[5] *Separable invariance* – when two or more separate segments in the same tone row are preserved.

Figure 2.6a – Pentachordal and dyadic invariance.

		5-27							5-27			
0	1	5	8	t	4	3	7	9	2	e	6	
e	0	4	7	9	3	2	6	8	1	T	5	
7	8	0	3	5	e	t	2	4	9	6	1	5-27
4	5	9	0	2	8	7	e	1	6	3	t	
2	3	7	t	0	6	5	9	e	4	1	8	
8	9	1	4	6	0	e	3	5	t	7	2	
9	t	2	5	7	1	0	4	6	e	8	3	
5	6	t	1	3	9	8	0	2	7	4	e	
3	4	8	e	1	7	6	t	0	5	2	9	
t	e	3	6	8	2	1	5	7	0	9	4	5-27
1	2	6	9	e	5	4	8	t	3	0	7	
6	7	e	2	4	t	9	1	3	8	5	0	

Figure 2.6b – Order Repositioning Pattern – 01234 → 13240



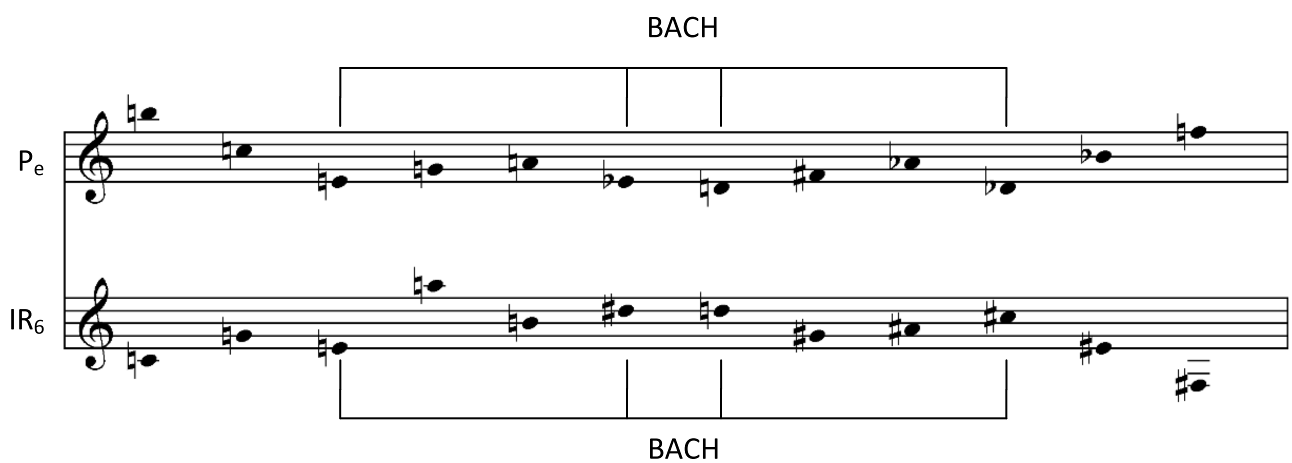
This type of invariance is particularly unique because of the shared properties to the hexachordal combinatoriality and its treatment of the BACH motive as will be discussed in the next chapter. The operations required for this type of pentachordal invariance are the exact same operations used to achieve hexachordal combinatoriality. Table 3 lists the operations needed to achieve this pentachordal invariance.

Table 2.3 – List of operations required for pentachordal invariance.

Tone Rows	Operation
$P_0 - I_7$	T_7I
$P_e - I_6$	T_5I
$P_7 - I_2$	T_9I
$P_4 - I_e$	T_3I
$P_2 - I_9$	T_eI
$P_8 - I_3$	T_eI
$P_9 - I_4$	T_1I
$P_5 - I_0$	T_5I
$P_3 - I_t$	T_1I
$P_t - I_5$	T_3I
$P_1 - I_8$	T_9I
$P_6 - I_1$	T_7I

If one is to retrograde one of the tone-rows that are paired together, a unique relationship is found regarding Dallapiccola's motivic material – the BACH motive. Example 2.2 shows a pair of tone-rows, where I_6 is retrograded, and the invariant BACH motive produced.

Example 2.2 – Tone-rows P_e and IR_6 shown with invariant BACH motive.



The BACH motive will be later discussed below in detail. We introduce it here to show that the BACH invariance overlaps with pentachordal, tetrachordal, and hexachordal invariance, all of which share the same list of operations.

Tetrachordal Invariance

In figure 2.7, the thinly-outlined tetrachord is shown as invariant at operation T_1I. Unlike the pentachord, the tetrachordal invariance occupies the same segment of the tone row, but not the same order positions. Notice that the first two dyads swap positions, and the last two dyads swap positions, creating double *order invariance*.[6] These are doubly invariant because they both preserve the pitch classes, the same segment of the tone row, and conform to a particular positioning pattern. Figure 2.7 shows three different sets of tetrachordal invariances and the order repositioning pattern.

Figure 2.7 – Tetrachordal invariance.

0	1	5	8	t	4	3	7	9	2	e	6
e	0	4	7	9	3	2	6	8	1	t	5
7	8	0	3	5	e	t	2	4	9	6	1
4	5	9	0	2	8	7	e	1	6	3	t
2	3	7	t	0	6	5	9	e	4	1	8
8	9	1	4	6	0	e	3	5	t	7	2
9	t	2	5	7	1	0	4	6	e	8	3
5	6	t	1	3	9	8	0	2	7	4	e
3	4	8	e	1	7	6	t	0	5	2	9
t	e	3	6	8	2	1	5	7	0	9	4
1	2	6	9	e	5	4	8	t	3	0	7
6	7	e	2	4	t	9	1	3	8	5	0

The thinly-outlined tetrachord is not invariant in every prime form row through a single operation (as the pentachord was). Rather, a different operation is needed for each tone row

[6] *Double Order Invariance* – a portion of a tone row which remains unchanged under a transformation (transpositionally and/or inversionally) and is not specific to the order position of the tone row, but will occupy the same portion of the tone row.

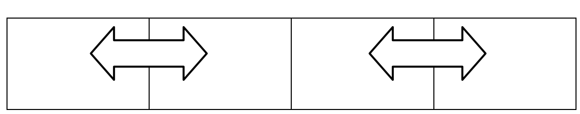
order to generate the invariant tetrachord (see table 2.4). Inversional relationships have a more subtle relationship than that of transpositional and therefore become more difficult to perceive aurally versus the transpositional invariances.

Note that there are two instances of each operation, further pairing the pairs of tone rows with the same operation. Table 2.4 shows the operations required to preserve the thinly-outlined tetrachord. The order repositioning pattern and the portion of the tone-row remain consistent for all three tetrachordal invariances. Unlike the pentachordal invariance, the tetrachordal invariance is always invariant at the first four order positions of each tone row. This characteristic allows for elision between two tone rows, especially when cross partitioning is used. Figure 2.8 shows the order repositioning of the tetrachords.

Table 2.4 – List of operations required for the thinly-outlined tetrachords.

Tone Rows	Operation
$P_0 – I_1$	T_1I
$P_e – I_0$	T_eI
$P_7 – I_8$	T_3I
$P_4 – I_5$	T_9I
$P_2 – I_3$	T_5I
$P_8 – I_9$	T_5I
$P_9 – I_t$	T_7I
$P_5 – I_6$	T_eI
$P_3 – I_4$	T_7I
$P_t – I_e$	T_9I
$P_1 – I_0$	T_3I
$P_6 – I_7$	T_1I

Figure 2.8 – Order Repositioning Pattern (Thinly-Outlined Tetrachords) – 0123 → 1032

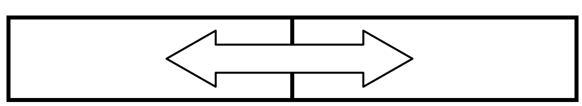


The shaded tetrachord is also invariant at various operations, but the operations now belong to what we shall call the *even stream invariance*[7], where *n* equals an even number. Note that there is, again, two of each operation, further pairing the pairs of tone rows with similar operations. Table 2.5 shows the operations required to preserve the shaded tetrachord. These operations pair the same prime form tone-rows that were previously paired in table 2.4.

Table 2.5 – List of operations required for the shaded tetrachord.

Tone Rows	Operation
$P_0 – I_8$	T_8I
$P_e – I_7$	T_6I
$P_7 – I_3$	T_tI
$P_4 – I_0$	T_4I
$P_2 – I_t$	T_0I
$P_8 – I_4$	T_0I
$P_9 – I_5$	T_2I
$P_5 – I_1$	T_6I
$P_3 – I_e$	T_2I
$P_t – I_6$	T_4I
$P_1 – I_9$	T_tI
$P_6 – I_2$	T_8I

Figure 2.9 – Order Repositioning Pattern (Shaded Tetrachord) – 0123 → 2301



The order repositioning pattern differs to that of the shaded tetrachord, but remains consistent throughout the various operations as well as the portion of the row – the first two dyads swap positions with the last two dyads, another case of double order invariance. These two tetrachordal invariances, then, are examples of double order invariance.

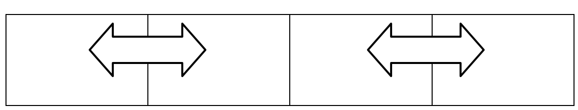
[7] *Even stream invariance* – a transformational operation where *n* equals an even number, T_n or T_nI.

The third tetrachord, which is thickly outlined (see figure 2.7), is also a case of double order invariance. This time the invariants will not be a discrete tetrachord of the row, and will occupy order positions 1 through 4. This tetrachord shares the same order repositioning pattern as the thinly-outlined tetrachord first discussed. Table 2.6 shows the operations used to preserve the thickly-outlined tetrachord which belong to the even stream operations.

Table 2.6 – A list of operations required for invariance of the thickly-outlined tetrachords.

Tone Rows	Operation
$P_0 – I_6$	T_6I
$P_e – I_5$	T_4I
$P_7 – I_1$	T_8I
$P_4 – I_t$	T_2I
$P_2 – I_8$	T_tI
$P_8 – I_2$	T_tI
$P_9 – I_3$	T_0I
$P_5 – I_e$	T_4I
$P_3 – I_9$	T_0I
$P_t – I_4$	T_2I
$P_1 – I_7$	T_8I
$P_6 – I_0$	T_6I

Figure 2.10 – Order Repositioning Pattern (Thinly-Outlined and Thickly-Outlined Tetrachord) – 0123 → 1032



This particular set of tetrachords is unique since the tetrachords are invariant within the same tone row. It should first be mentioned that when one tetrachord from either group is paired with a tetrachord from the other, the relationship is already accounted for through the previously discussed invariances. Furthermore, these invariances are still considered double order invariance because they occupy the same segment of the tone-row. However, the tetrachords do not share the same order positions. Figure 2.11 highlights this type of separable invariance.

Figure 2.11 – Separable tetrachordal invariance.

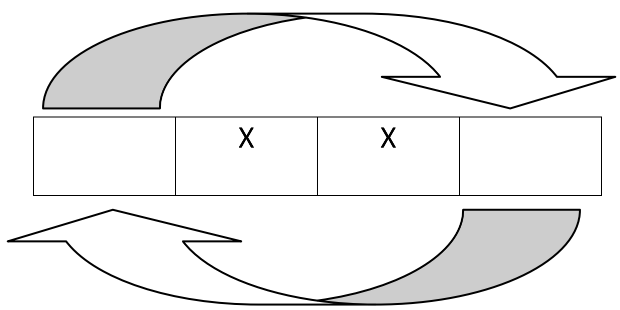
						3-3						
0	1	5	8	t	4	3	7	9	2	e	6	
e	0	4	7	9	3	2	6	8	1	t	5	
7	8	0	3	5	e	t	2	4	9	6	1	
4	5	9	0	2	8	7	e	1	6	3	t	
2	3	7	t	0	6	5	9	e	4	1	8	
8	9	1	4	6	0	e	3	5	t	7	2	
9	t	2	5	7	1	0	4	6	e	8	3	3-3
5	6	t	1	3	9	8	0	2	7	4	e	
3	4	8	e	1	7	6	t	0	5	2	9	
t	e	3	6	8	2	1	5	7	0	9	4	
1	2	6	9	e	5	4	8	t	3	0	7	
6	7	e	2	4	t	9	1	3	8	5	0	

In figure 2.11 I have also shaded the pitch classes separating the invariant tetrachords. Though the pitch classes of the trichord are not invariant, the intervallic content is – sustaining the middle trichord as a 3-3. The 3-3 invariance is a general phenomenon in any given series, but it is interesting that it is preserved along with two common tones. Table 2.7 shows the operations required to maintain separable invariance. This type of invariance belongs to the odd stream operation group. Even and odd stream operation groups are used only to categorize the transformational patterns that emerge from the chordal invariances.

Table 2.7 – A list of operations for the separable tetrachordal invariance.

Tone Rows	Operation
$P_0 - I_7$	T_7I
$P_e - I_6$	T_5I
$P_7 - I_2$	T_9I
$P_4 - I_e$	T_3I
$P_2 - I_9$	T_eI
$P_8 - I_3$	T_eI
$P_9 - I_4$	T_1I
$P_5 - I_0$	T_5I
$P_3 - I_t$	T_1I
$P_t - I_5$	T_3I
$P_1 - I_8$	T_9I
$P_6 - I_1$	T_7I

Figure 2.12 – Order Repositioning Pattern (Shaded and Non-Shaded Tetrachords) – 0123→ 3120



These invariant relationships between pitch class content and intervallic content create a heightened sense of similarity between related tone rows. Dallapiccola is able to make eleven out of the possible twelve elements of the tone row relatable through either interval invariance or pitch-class invariance.

This last set of tetrachordal invariance discussed above appears twice as often as the previous. Each tetrachord could be related to the other three tetrachords through a transformational operation. However, these relationships have already been discussed in figure 2.7, except they were discussed as two different invariances associated with the other two tetrachords, not as a single tetrachord that is invariant at three different levels. Figure 2.13 highlights the four invariant tetrachords labeled "a" through "d" accordingly.

Figure 2.13 – Four invariant tetrachords.

	0	1	5	8	t	4	3	7	9	2	e	6	c
	e	0	4	7	9	3	2	6	8	1	**t**	5	
	7	8	0	3	5	e	t	2	4	9	**6**	1	
	4	5	9	0	2	8	7	e	**1**	**6**	**3**	**t**	d
	2	3	7	t	0	6	5	9	e	4	**1**	8	
a	8	9	1	4	6	0	e	3	5	t	7	2	
	9	t	2	5	7	1	0	4	6	e	8	3	
	5	**6**	**t**	**1**	**3**	9	8	0	2	7	4	e	
b	**3**	4	8	e	1	7	6	t	0	5	2	9	
	t	e	3	6	8	2	1	5	7	0	9	4	
	1	2	6	9	e	5	4	8	t	3	0	7	
	6	7	e	2	4	t	9	1	3	8	5	0	

The prime form tetrachord (labeled as "a") in tone-row P_5 will have three different sets of relationships, one set for each other tetrachord. Table 2.8 shows the operations required in preserving the tetrachord. In order to find the reverse relationship (tetrachord "x" to "a"), the complementary operation is used. The relationship between tetrachords "a" to "b" follows the odd number steam operations and the relationship between tetrachords "a" to "c" follows the even number stream operations. The relationship between tetrachords "a" to "d" does not follow an even or odd stream operational group because it is not related by inversion. This relationship will have a single T_n operation along with and an equal and opposite operation. In this case, the “a” to “d” relationship is described as T_e, and its equal and opposite operation is T_1.

Table 2.8 – Tetrachordal invariance for tetrachords “a” through “d”.

a-b		a-c		a-d	
Tone Rows	Operation	Tone Rows	Operation	Tone Rows	Operation
$P_0 – I_7$	T_7I	$P_0 – I_6$	T_6I	$P_0 – P_e$	T_e
$P_e – I_6$	T_5I	$P_e – I_5$	T_4I	$P_e – P_t$	T_e
$P_7 – I_2$	T_9I	$P_7 – I_1$	T_8I	$P_7 – P_6$	T_e
$P_4 – I_e$	T_3I	$P_4 – I_t$	T_2I	$P_4 – P_3$	T_e
$P_2 – I_9$	T_eI	$P_2 – I_8$	T_tI	$P_2 – P_1$	T_e
$P_8 – I_3$	T_eI	$P_8 – I_2$	T_tI	$P_8 – P_7$	T_e
$P_9 – I_4$	T_1I	$P_9 – I_3$	T_0I	$P_9 – P_8$	T_e
$P_5 – I_0$	T_5I	$P_5 – I_e$	T_4I	$P_5 – P_4$	T_e
$P_3 – I_t$	T_1I	$P_3 – I_9$	T_0I	$P_3 – P_2$	T_e
$P_t – I_5$	T_3I	$P_t – I_4$	T_2I	$P_t – P_9$	T_e
$P_1 – I_8$	T_9I	$P_1 – I_7$	T_8I	$P_1 – P_0$	T_e
$P_6 – I_1$	T_7I	$P_6 – I_0$	T_6I	$P_6 – P_5$	T_e

Again, each stream of operations has two of each T_nI operation. These are paired in the same fashion as they were in the previous tetrachords, pentachords, and hexachords as shown in table 2.8.

Trichordal Invariance

The trichordal invariance found in the series is both separable and double order invariance, occurring at the same time. The invariants occupy the same portion of the tone-row but the invariants do not remain in the subsequent tone row. This is a type of separable invariance because it is separated under a transformation, and is also double order invariance because it occupies the same order positions of the tone-row, even though it appears in a different tone-row. Figure 2.14 highlights the trichordal invariances. It is a general phenomenon of any given twelve-tone series that if the invariance is present in the prime forms, it will also be present in the inversional forms of the tone row. Therefore, the invariance that appears in the prime forms will also appear in the inversional forms, but with an equal and opposite operation.

Figure 2.14 – Trichordal invariance.

0	1	5	8	T	4	3	7	9	2	e	6
e	0	4	7	9	3	2	6	8	1	t	5
7	8	0	3	5	e	t	2	4	9	6	1
4	5	9	0	2	8	7	e	1	6	3	t
2	3	7	t	0	6	5	9	e	4	1	8
8	9	1	4	6	0	e	3	5	t	7	2
9	t	2	5	7	1	0	4	6	e	8	3
5	6	t	1	3	9	8	0	2	7	4	e
3	4	8	e	1	7	6	t	0	5	2	9
t	e	3	6	8	2	1	5	7	0	9	4
1	2	6	9	E	5	4	8	t	3	0	7
6	7	e	2	4	t	9	1	3	8	5	0

First note that the relationship between the shaded and the thinly-outlined trichords is the same as the relationship between the thickly-outlined and dashed-outlined trichords. Though the trichords first occur in the same tone row, they will split and become independently invariant in other tone rows. Transformations are constant and are, therefore, complementary when applied to the inversional row forms. Table 2.9 shows the operations required to preserve the trichords.

Table 2.9 – A list of operations required for trichordal invariance.

Tone Row	Operation/Tone Row (Outlined Trichord)	Operation/Tone Row (Shaded Trichord)
P_0	T_1/P_1	T_e/P_e
P_e	T_1/P_0	T_e/P_t
P_7	T_1/P_8	T_e/P_6
P_4	T_1/P_5	T_e/P_3
P_2	T_1/P_3	T_e/P_1
P_8	T_1/P_9	T_e/P_7
P_9	T_1/P_t	T_e/P_8
P_5	T_1/P_6	T_e/P_4
P_3	T_1/P_4	T_e/P_2
P_t	T_1/P_e	T_e/P_9
P_1	T_1/P_2	T_e/P_0
P_6	T_1/P_7	T_e/P_5

As just mentioned, trichords which are invariant in the inversional form of the tone-row have reciprocal relationships to those in the prime form tone-rows. Table 2.10 shows the operations required to preserve the trichords in the inversional row form. The order positioning pattern is the same for both sets of invariant trichords, but are opposite within each set.

Table 2.10 – A list of operations required for trichordal invariance.

Tone Row	Operation/Tone Row (Thickly Outlined Trichord)	Operation/Tone Row (Dashed Trichord)
I_0	T_e/I_e	T_1/I_1
I_1	T_e/I_t	T_1/I_0
I_5	T_e/I_6	T_1/I_8
I_8	T_e/I_3	T_1/I_5
I_t	T_e/I_1	T_1/I_3
I_4	T_e/I_7	T_1/I_9
I_3	T_e/I_8	T_1/I_t
I_7	T_e/I_4	T_1/I_6
I_9	T_e/I_2	T_1/I_4
I_2	T_e/I_9	T_1/I_e
I_e	T_e/I_0	T_1/I_2
I_6	T_e/I_5	T_1/I_7

Figure 2.15a – Order Positioning Pattern (Thinly-Outlined and Thickly-Outlined Trichord) – 012 → 201

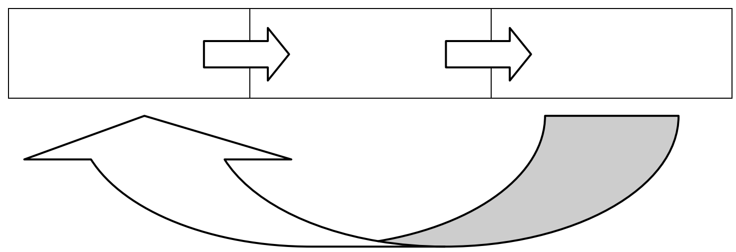
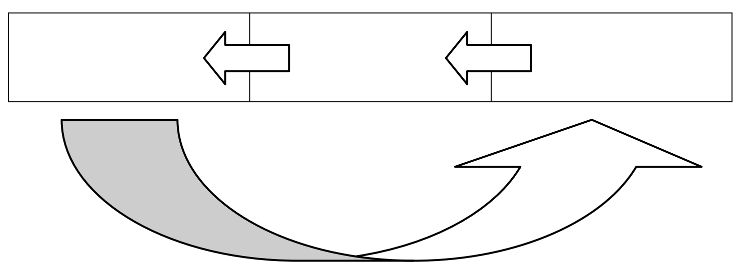


Figure 2.15b – Order Positioning Pattern (Shaded and Dashed-Outlined Trichord) – 012 → 120



Non-Segmental Invariance

One last type of invariance must still be discussed, in regards to the motivic material. The BACH motive serves as thematic material throughout the entire *Quaderno*. Dallapiccola pays particular attention to the tetrachord in his music, and pays close attention to it with respect to the series construction. Non-segmental invariance can be defined as the preservation of an element of the series that does not appear as a segmental portion of the series. The invariance of the BACH motive relates the prime forms and the inversion-retrograde forms of the tone row. The BACH motive is also invariant at the inversional and the retrograde tone row forms of the series. Though similar in nature, these two invariances are not the same, but can be both described as double order invariance because they occupy the same order positions of the tone row amongst themselves. The most striking characteristic of this invariance is its order positioning pattern. In the prime and inversional tone rows forms, the BACH motive appears in its proper order, but when put under a transformation for invariance, the BACH motive appears in its retrograde form. Since the BACH motive serves as thematic material throughout the *Quaderno*, it is expected to appear in its original form, as well as variant forms. Figure 2.16 highlights the BACH motive invariances.

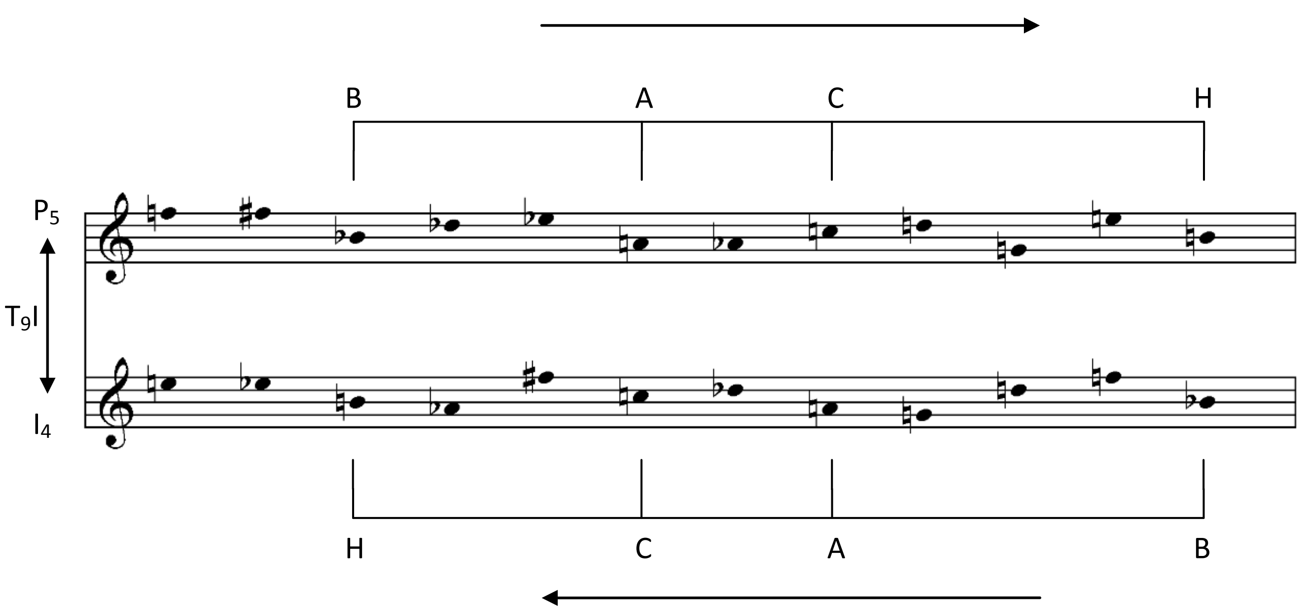
Figure 2.16 – Non-segmental invariance of the BACH motive.

				B			A		C				H	
		0	1	2	3	4	5	6	7	8	9	t	e	
B	0	0	1	5	8	t	4	3	7	9	2	e	6	
A	1	e	0	4	7	9	3	2	6	8	1	t	5	
	2	7	8	0	3	5	e	t	2	4	9	6	1	H
	3	4	5	9	0	2	8	7	e	1	6	3	t	
C	4	2	3	7	t	0	6	5	9	e	4	1	8	
	5	8	9	1	4	6	0	e	3	5	t	7	2	C
	6	9	t	2	5	7	1	0	4	6	e	8	3	
	7	5	6	t	1	3	9	8	0	2	7	4	e	A
	8	3	4	8	e	1	7	6	t	0	5	2	9	
	9	t	e	3	6	8	2	1	5	7	0	9	4	
H	t	1	2	6	9	e	5	4	8	t	3	0	7	
	e	6	7	e	2	4	t	9	1	3	8	5	0	B
		H	C			A						B		

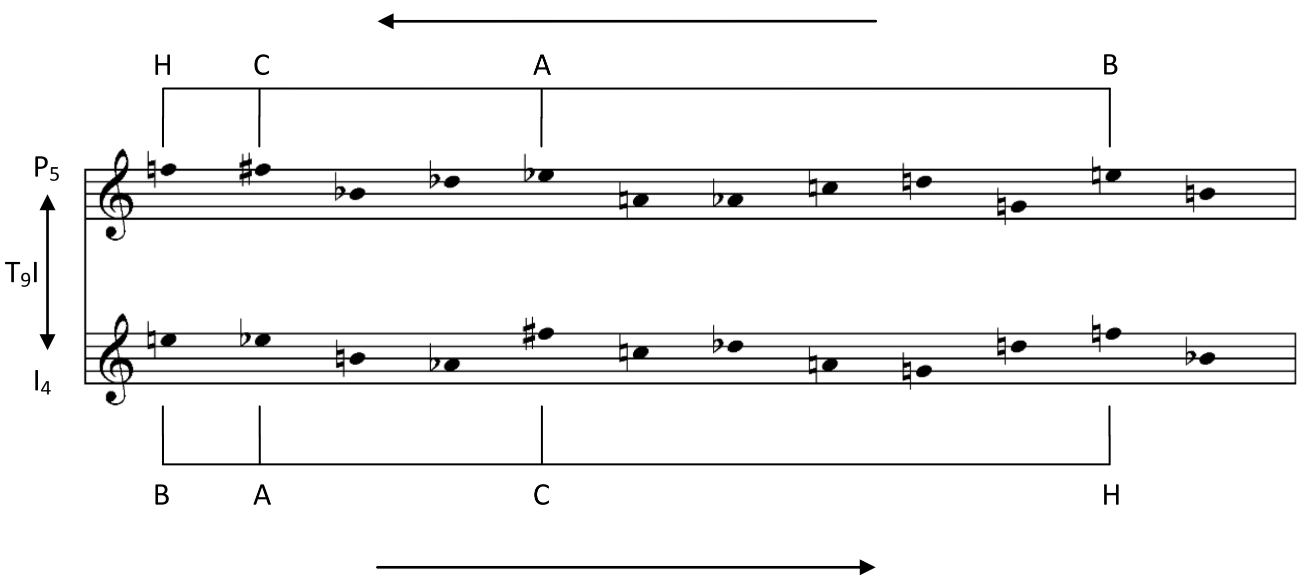
The BACH motive can be found in the prime form at order positions <2 5 7 e>, inversion form at <0 1 4 t>, retrograde form at <0 1 4 t>, and retrograde-inversion at <2 5 7 e>. Notice that the motive in the prime form and in the retrograde-inversion occupies the same space; this is why it is double order invariance. The same relationship occurs between the inversion and retrograde forms.

In the case of the BACH motive, invariance has a different meaning. Similarly to double order invariance, The BACH motive remains unchanged under a transformation and will also occupy the same portion of the row. However, because the relationship of invariance is related through retrograded tone rows, the BACH motive will always appear in its retrograde, but still will maintain the order, either B-A-C-H or H-C-A-B, and the same position. Though this relationship can be observed in the matrix, it is more easily seen as notes on a staff, Example 2.3a and 2.3b shows tone rows P_5 and I_4 and the BACH motive invariance, (compare to example 2.2).

Example 2.3a – Non-segmental BACH invariance.



Example 2.3b – Non-segmental BACH invariance



It is interesting that (as observed in example 2.3a and b) the same operation is used for the same pair of tone-rows where the BACH motive is invariant. Table 2.11 shows the required operations to the non-segmental invariance.

Table 2.11 – A list of operations required for the BACH invariance.

Tone Rows	Operation	Tone Rows	Operation
$P_0 - I_e$	T_eI	$R_0 - I_e$	T_eI
$P_e - I_t$	T_9I	$R_e - I_t$	T_9I
$P_7 - I_6$	T_1I	$R_7 - I_6$	T_1I
$P_4 - I_3$	T_7I	$R_4 - I_3$	T_7I
$P_2 - I_1$	T_3I	$R_2 - I_1$	T_3I
$P_8 - I_7$	T_3I	$R_8 - I_7$	T_3I
$P_9 - I_8$	T_5I	$R_9 - I_8$	T_5I
$P_5 - I_4$	T_9I	$R_5 - I_4$	T_9I
$P_3 - I_2$	T_5I	$R_3 - I_2$	T_5I
$P_t - I_9$	T_7I	$R_t - I_9$	T_7I
$P_1 - I_0$	T_1I	$R_1 - I_0$	T_1I
$P_6 - I_5$	T_eI	$R_6 - I_5$	T_eI

Once again, under each list of operations there are two kinds of each T_nI operation in the odd stream operation group.

Though the application of set-theory analysis offers the tools to describe the basic concepts of Dallapiccola's music, it is necessary to look beyond the basic relationships and interpret Dallapiccola's compositional choices. I begin with one of Dallapiccola's most often used twelve-tone devices – the cross partition - followed by a discussion of polarity in twelve-tone music.

Cross Partitioning

The choice to follow Brian Alegant's recent theories on cross partitioning stems from the following passage from Dallapiccola's essay from the *Music Survey* (1951):

> "Before reaching this rhythmic and melodic definition of the series, we may find it compressed into a single chord of twelve notes, two chords of six notes, three of four, four of three notes, or even six two-note chords….to speak only of the most elementary possibilities. It will be understood that, in every such combination, the sense of *polarity* must be alive and present, so as to enable the listener to follow the musical argument."[8]

Though Dallapiccola does not explicitly acknowledge cross partitioning, his description of compressed notes implies the four types of twelve note configurations described by Alegant. Cross partitioning is a twelve-tone compositional device which is used not only to control the horizontal elements of the music, but also the vertical elements of a twelve-tone design. The basic idea of cross partitioning is to isolate non-segmental pitches from a series, extracting

[8] Luigi Dallapiccola, "On the Twelve-Tone Road", *Music Survey 4* (1951): 329.

pitches that do not appear in sequence in the twelve-note series. Commonly, linear presentations are found in a twelve-note series which is often evenly sub-divided into hexachords, tetrachords, trichords, or dyads – known as partitioning. The restriction of linear presentations is that it is limited to the discrete chords of the series and a specific order, not allowing any room for rearrangement. However, cross partitioning breaks loose from the strict ordering and presents a twelve-note series in a more aggregate fashion, versus a linear presentation. As Alegant defines it, a cross partition is a "two dimensional configuration of pitch classes whose columns are realized as chords, and whose rows are differentiated from one another by registral, timbral, or other means."[9] With the use of all twelve pitches, one can make only four types of "even" configurations, as Dallapiccola and Alegant claim. These are called "even" cross partitions because they conform to rectangular configurations. Twelve pitches can only be configured into four types of even configurations - two by six pitches, three by four pitches, four by three pitches, and six by two pitches. All of which were dicussed in Dallapiccola's essay from *Music Survey* (1951), as quoted above. Other types of configurations that do not conform to one of the even configuration are considered to be "uneven". On the other hand, a linear presentation, delivers what it promises – a straight-up line of the twelve pitches. For example the hypothetical twelve-note series: <0 1 2 3 4 5 6 7 8 9 t e> can be subjected to 4^3 cross partition, where the first integer represents the number of vertical elements and the exponent represents the number of horizontal elements of the configuration. Figure 2.17 shows what a 4^3 cross partition could look like.

[9] Brian Alegant, "Cross-Partitions as Harmony and Voice Leading in Twelve-Tone Music" *Music Theory Spectrum* Vol. 23, No. 1 (2001): 1.

Figure 2.17 – Hypothetical twelve-tone series <0 1 2 3 4 5 6 7 8 9 t e>.

4^3

0	4	8	→ Melody
1	5	9	
2	6	t	
3	7	e	

Chords ↓

The vertical elements would be perceived as chords and the horizontal elements would be perceived as melodic content. Example 2.4 shows an example of an "even" 4^3 configuration, taken from the opening measures of Schoenberg's *Klavierstück Op.33a.*

Example 2.4 – Schoenberg's *Klavierstück Op. 33a.*

Schoenberg's entire twelve-note series is presented in the first measure through the presentation of three tetrachords. The analyst is able to only find a partial ordering of the series in four-note clusters and cannot determine which notes within each tetrachord occurs first. This concept will be discussed in a later chapter, but for now please note the aggregate presentation

– one cannot define the ordering of the series because the notes are stacked as chords. It is not until several measures later where Schoenberg gives an official ordering on the pitches. After an official ordering of the series is explicit, the analyst will then be able to identify tone rows and tone-row transformations.

Alegant refers to a set of four different “even” cross partition configurations – 6^2, 4^3, 3^4, and 2^6. This first number represents the number of vertical elements while the number in superscript represents the number of columns. These are considered “even” cross partitions because they conform to perfect rectangular designs.[10]

[10] Brian Alegant, *The Twelve-Tone Music of Luigi Dallapiccola* (Rochester: University of Rochester Press, 2010), 21.

Figure 2.18 – Four types of cross partitioning configurations.

6^2

*	*
*	*
*	*
*	*
*	*

4^3

*	*	*
*	*	*
*	*	*
*	*	*

3^4

*	*	*	*
*	*	*	*
*	*	*	*

2^6

*	*	*	*	*	*
*	*	*	*	*	*

Once a configuration is made, a composer could subject it to the "slot-machine" transformation. The "slot-machine" transformation allows for the vertical elements to be permuted within each of their columns. By doing this, each column will preserve the same vertical sonorities, but will alter the horizontal elements, therefore, maintaining the configuration's harmonies, but varying the melody. The vertical elements are at liberty to move about within each of their own column, thus like a slot machine.[11] Figure 2.19 are some possible examples of how a twelve-note series could be subjected to the "slot-machine" technique, each time jumbling the vertical elements within their own columns.

Figure 2.19 – Various permutations of the hypothetical series.

4^3

0	3	6	9	0	5	6	e	2	4	7	9	1	3	6	t
1	4	7	t	2	3	7	t	0	3	6	t	2	5	7	9
2	5	8	e	1	4	8	9	1	5	8	e	0	4	8	e

These types of configurations will be applied to the three contrapuntal movements of the *Quaderno* in the analysis chapter, some that are conforming to "even" cross partitions,

[11] Brian Alegant, *The Twelve-Tone Music of Luigi Dallapiccola* (Rochester: University of Rochester Press, 2010), 21.

others creating “uneven” cross partitions. The term “uneven” is used because the pitches do not conform to any of the rectangular configurations. An “uneven” cross partition does not have a set number of columns or a set number of elements within each column.

Polarity

An excerpt from Dallapiccola’s 1951 essay describing how he came upon the twelve-note system from the *Music Survey* shows his conception of achieving centricity through *polarity*. He describes it as follows:

> Thus I came to the conclusion that if, in the twelve-note system, the tonic had disappeared, taking with it the tonic-dominant relationship, and if, in consequence, sonata form had completely disintegrated, there still existed, nevertheless, a power of attraction, which I will call *polarity* (I do not know whether such a definition has been used before, or whether there is another) : I mean by this term the extremely subtle relationships which exists between certain notes. These relationships are not always easily perceptible today, being much less obvious than that of tonic to dominant but they are there, all the same. The interesting point about this polarity is the fact that it can change (or be changed) from one work to another. One series can reveal to us *polarity* that exists between the first and twelfth sounds ; another that which exist between the second and the ninth ; and so on.[12]

Dallapiccola is suggesting that *polarity* could serve the twelve-tone system in a way the tonic-dominant relationship serves tonality. The tonic-dominant relationship in tonal music creates a sense of attraction centred on a single pitch (i.e. the tonic). This sense of attraction modulates throughout single compositions via transitional and developmental stages in a sonata. Dallapiccola suggests that *polarity* also has the ability and potential to “modulate” amongst centric ideas, analogous of tonality where tonic keys can modulate to other related keys such as

[12] Luigi Dallapiccola, "On the Twelve-Tone Road", *Music Survey 4* (1951): 325.

dominant, relative minor, or parallel minor key areas. Straus makes similar observations in his *Introduction to Post-Tonal Theory (2005)* in his analysis of Béla Bartók's *Sonata for Two Pianos and Percussion* – "The twelve inversional axes have the potential to function like the twelve major/minor keys of traditional tonality, including the possibility that music might "modulate" from one axis to another."[13] In response to the above quotation, David Mancini's article *Twelve-Tone Polarity in Late Works of Luigi Dallapiccola (1986)* from the *Journal of Music Theory* raises several important questions:

> "What specific methods might create this attractive force in music that, by definition, denies any feeling of tonic? What is the nature of these "subtle relationships" between pitches? Are the relationships inherent in some specific ordering of the tonal chromatic or are they *contextually* imposed? And if polarity can indeed exist in twelve-tone music, then is "tonic" a pitch class, an interval, or a pitch-class set?"[14]

Mancini goes on to suggest through musical excerpts that "tonic" can indeed be established as an interval class or pitch-class set. Many of Mancini's examples of polarity involve the isolation of two adjacent pitch classes being combined with other adjacent pitch classes in order to create a centric pitch-class set. This is what I referred to earlier as isolating non-segmental elements of the series. Mancini's examples support only his thoughts on "tonic" as a pitch-class set. Through a collection of data, I argue that Dallapiccola achieves a sense of polarity through both a pitch class and an interval class. Based on the frequency of an interval class and the timing in which it arrives, I postulate that Dallapiccola emphasizes certain interval classes as a

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[13] Straus, Joseph. *Introduction to* (New Jersey: Prentice Hall, 2005), 137.
[14] David L. Mancini, "Twelve-Tone Polarity in Late Works of Luigi Dallapiccola", *Journal of Music Theory,* Vol. 30, No. 2 (1986): 204.

centric aspect of a movement. Dallapiccola also establishes centricity by an axis of symmetry through inversion.

ANALYSIS

N. 3 – Contrapunctus Primus

This chapter of analysis begins with Dallapiccola's first contrapuntal movement from the *Quaderno* - *Contrapunctus Primus.* After a brief introduction of tone-row identification, I explore the subtle relationships between tone rows and the qualities which connect them together. Dallapiccola's use of hexachordal and trichordal partitioning as well as cross partitioning are examined to reveal Dallapiccola's compositional organization. The use of the BACH motive is introduced in its original form, found in *Simbolo*, then is discussed as a motive undergoing transformation. Lastly,

Tone-Row Realization

Contrapunctus Primus (hereafter referred to as *Primus*) is the first contrapuntal work to appear in the *Quaderno*, offering a refreshing aural pallet change from the contrasting denser movements preceding it. The linear presentation of the tone row finally gives the listener, and the analyst, an official ordering of the series. Growing in complexity, *Primus* unfolds in three canonic sections. The first section, section A (mm. 1-8), consists of a two-voice canon at the unison and employs tone rows P_e and R_1. Section B (mm. 9-13) adds a middle voice creating a contrary harmonic relationship with the lower voice and a rhythmically separated contrary relationship with the upper voice as the two outer voices maintain their canonic relationship at the unison. In section B, the outer voices sustain tone row IR_6 while the newest middle voice presents R_4. The last section of the movement, section C (mm. 14-18), becomes increasingly more intricate with the subtle addition of a fourth voice. The fourth voice takes the same tone-row, now P_2, maintaining the canon in unison with the upper voice. The other two voices are also in canon by unison through the use of I_t. Subsequently, each of the voices in the two unison canons further create an intricate web of relationships with each of the other three voices, making section C the most dense and complex, and arguably the climax, of the movement. Table 3.1 is a realized representation of the tone rows achieved in *Primus*. Tone rows labeled with an arrow are related as literal complements.

Table 3.1 – Tone-row realization of *Primus*.

	mm. 1-5	**mm. 5-9**	**mm. 9-13**	**mm. 13-18**
	Section A		**Section B**	**Section C**
Voice 1	e 0 4 7 9 3 2 6 8 1 t 5 P_e	7 0 3 t 8 4 5 e 9 6 2 1 R_1	0 7 4 9 e 3 2 8 t 1 5 6 IR_6	2 3 7 t 0 6 5 9 e 4 1 8 P_2
Voice 2	e 0 4 7 9 3 2 6 8 1 t 5 P_e	7 0 3 t 8 4 5 e 9 6 2 1 R_1	0 7 4 9 e 3 2 8 t 1 5 6 IR_6	2 3 7 t 0 6 5 9 e 4 1 8 P_2
		Voice 3	t 3 6 1 e 7 8 2 0 9 5 4 R_4	t 9 5 2 0 6 7 3 1 8 e 4 I_t
			Voice 4	t 9 5 2 0 6 7 3 1 8 e 4 I_t

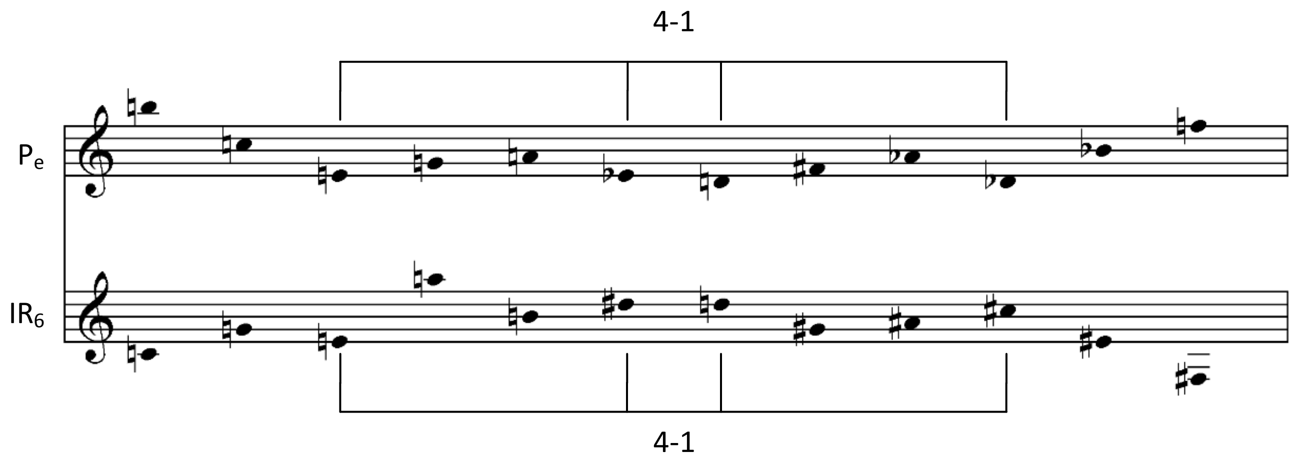
Dallapiccola weaves together the sections of *Primus*, through his tone-row selection. It is observed in the above table that section A and section C are divided independently by employing tone-rows that are literal complements of one another, marked by arrows. However, Dallapiccola achieves a connection between each section through a careful selection of tone-rows. The relationship between P_e and IR_6 was discussed in the pentachordal invariance section of chapter two. Figure 3.1 highlights the pentachordal relationship between P_e and IR_6 and the two common tones which separate the chordal invariance.

Figure 3.1 – Pentachordal invariance in *Primus.*

		5-27							5-27			
0	1	5	8	t	4	3	7	9	2	e	6	
e	0	4	7	9	3	2	6	8	1	t	5	
7	8	0	3	5	e	t	2	4	9	6	1	5-27
4	5	9	0	2	8	7	e	1	6	3	t	
2	3	7	t	0	6	5	9	e	4	1	8	
8	9	1	4	6	0	e	3	5	t	7	2	
9	t	2	5	7	1	0	4	6	e	8	3	
5	6	t	1	3	9	8	0	2	7	4	e	
3	4	8	e	1	7	6	t	0	5	2	9	
t	e	3	6	8	2	1	5	7	0	9	4	5-27
1	2	6	9	e	5	4	8	t	3	0	7	
6	7	e	2	4	t	9	1	3	8	5	0	

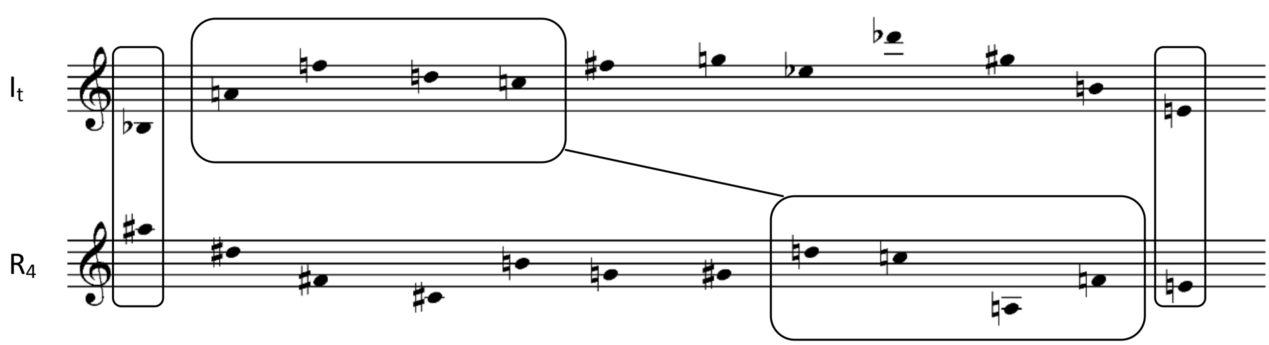
Dallapiccola's choice to use tone-rows P_e and IR_6 is an example of separable invariance by inversion. Of course, this relationship could also be described as being hexachordally I-combinatorial. However, because Dallapiccola uses the retrograded form of I_6, combinatoriality is never actually achieved, but a motivic element is preserved through non-segmental invariance. Example 3.1 shows non-segmental invariance at order positions <1458>.

Example 3.1 – Non-segmental invariance in *Primus.*



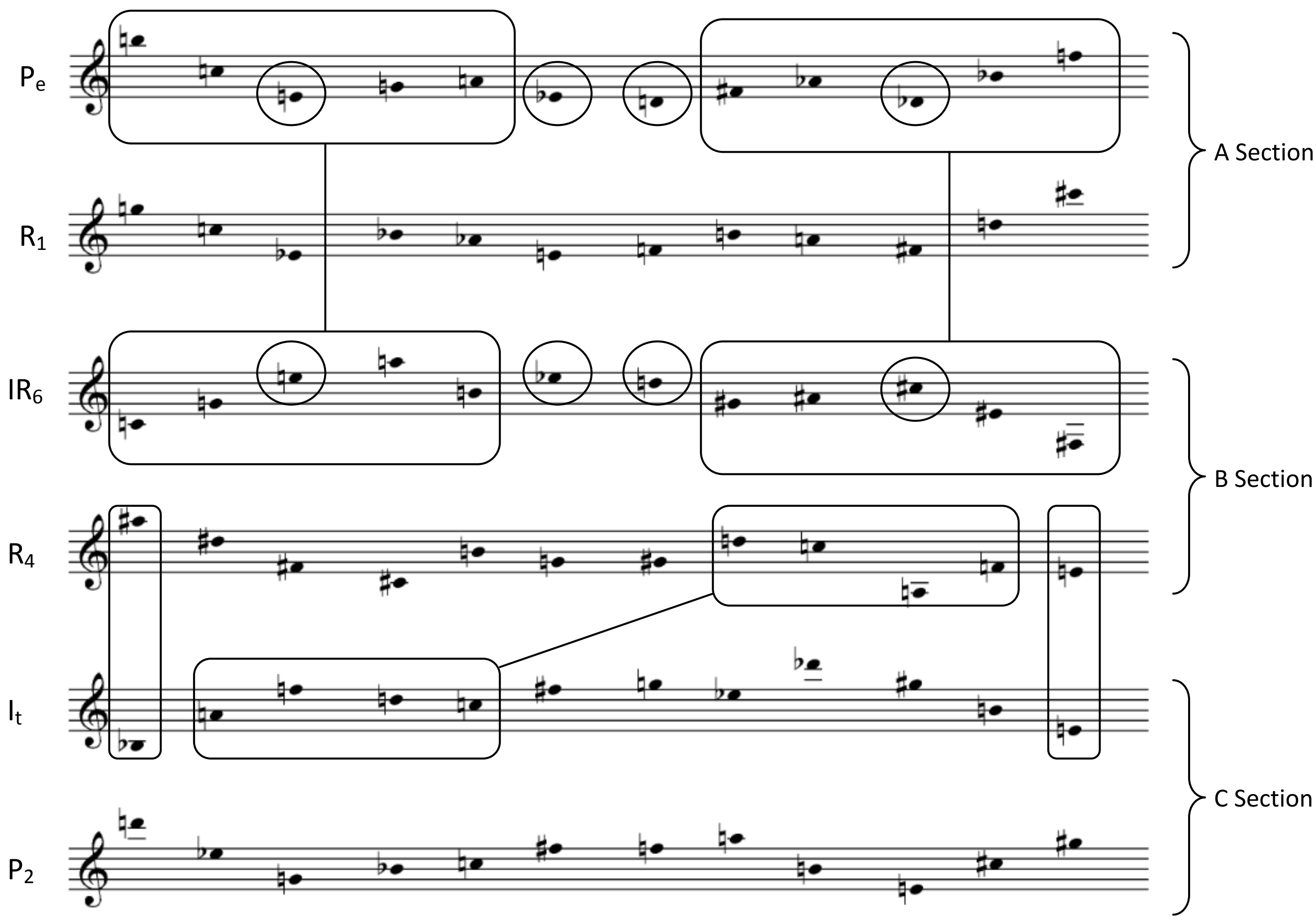
Recalling the BACH motive, Dallapiccola will always pay close attention to this tetrachord and treats it very carefully. This non-segmental invariance of the BACH motive shows how Dallapiccola exploits tone rows that have similar intervallic content. A similar relationship occurs between tone-rows R_4 and I_t, which ties together the B section to the C section. Example 3.2 shows the separable invariance and the non-segmental invariance in these two tone-rows.

Example 3.2 – Tone-rows I_t and R_4 from *Primus.*



This web of relationships demonstrates how Dallapiccola intricately weaves the tone-rows between sections together. Example 3.3 is a summary of all the tone-rows employed in *Primus*, organized by which section they appear in and by the properties that are shared between the given tone-rows. These relationships show that Dallapiccola's tone-row selection was not arbitrarily decided. The way in which the tone-rows are related to one another is analogous to related key areas in tonal music. Example 3.3 also summarizes the shared properties crossed between the three sections of the piece.

Example 3.3 – Tone-row relationships in *Primus.*



Example 3.3 also summarizes the shared properties crossed between the three sections of the piece.

Contour

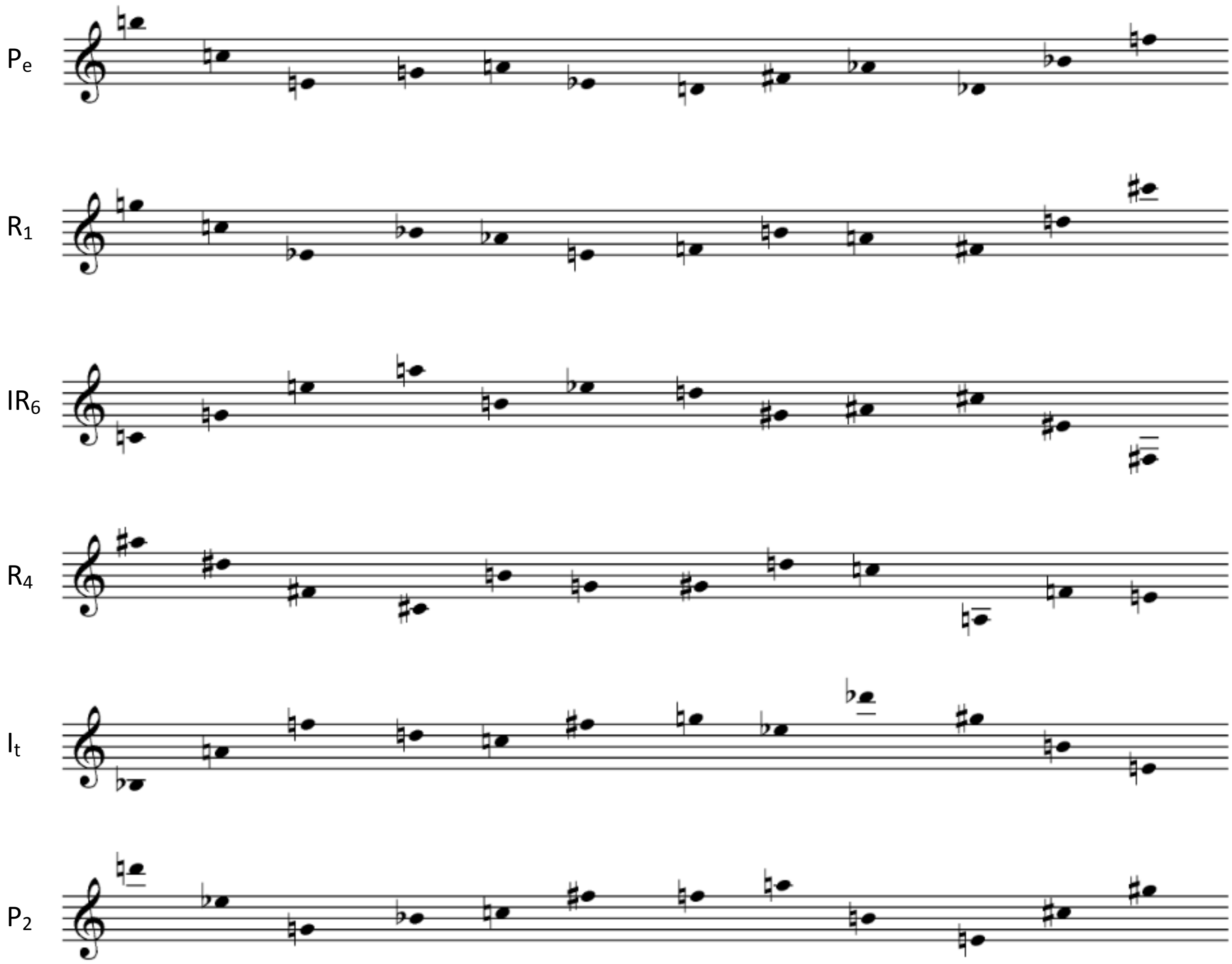
Dallapiccola reveals the complexity of his style through a multitude of techniques. As the tone rows employed in this movement have already been discussed, a more in depth look at the relationships among the tone rows is in order. The first two tone rows to appear are P_e and R_1. The most remarkable characteristics between these two tone rows are their uncanny contour similarity. It is not the choice of which tone row Dallapiccola decides on, but the form (retrograde) and the pitch arrangement that becomes most interesting. Example 3.4 demonstrates the contour similarities between the prime form and the retrograde form.

Example 3.4 – Tone rows P_e and R_1, as they appear in *Primus*.

In a carefully constructed series the similarities in contour remain through all combinations of row forms (prime, retrograde, inversion, and retrograde inversion). However, this contour characteristic can be, in any given two forms, in contrary or similar motion with one another. Notice in example 3.4 that P_e and R_1 do not contain the exact ordered pitch class intervals, but do have the same overall contour. In some instances the rows do not maintain the same type of contour (contrary or similar motion), however, overall it can be said that they possess similar contours.

This is true for the prime and retrograde forms, it must also be true for the inversion and retrograde inversion forms, as this is a generic property of a series. Since the prime and inversion forms are related through inversion, their contour relation will not result in similar motion, but instead contrary motion. Again, this is true for the prime and inversion forms, it will also be true in the retrograde and retrograde inversion forms. This particular characteristic can only be brought out through carefully thought out registral choices. In order to generate this type of contour relationship, Dallapiccola must follow a single consistent rule of symmetry in constructing the prime form. This means that the contour from the first half of the tone row must be reflected in the second half of the tone row so that retrograde forms of the row will reveal similar contour to the prime form. These contour similarities cannot exist if Dallapiccola arranged the prime form contour asymmetrically. It is only because of the symmetrical quality of the arranged prime form that these contour relationships can exist. In example 3.6, one can observe that similar contour relationships exist between the prime and retrograde forms and the inversion and retrograde-inversion forms.

Example 3.6 - Tone rows P_e, R_1, IR_6, and I_t are shown as they appear in *Primus.*

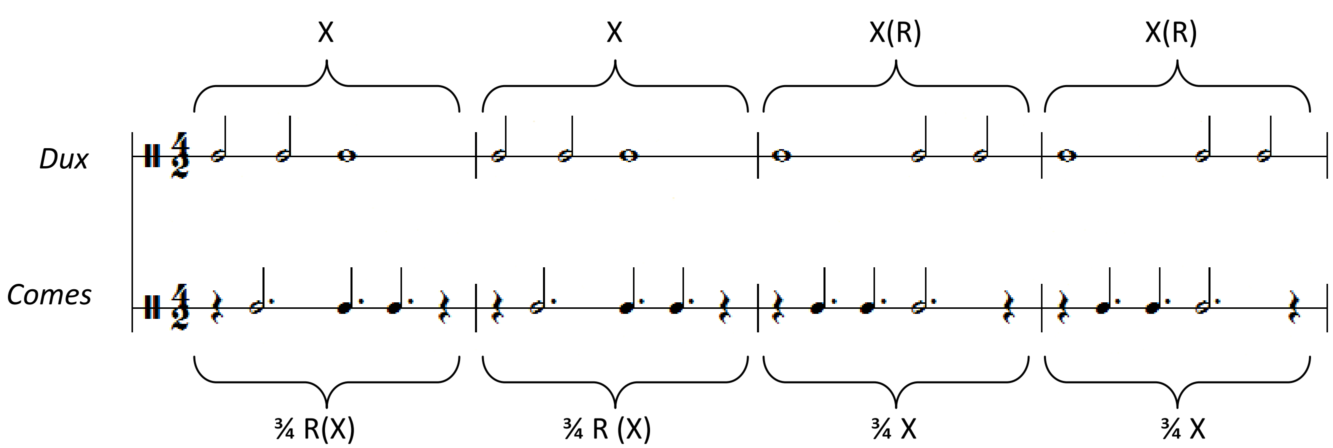


Dallapiccola's series, or any given twelve-tone series, is only a tool in which pitch classes are provided. It is up to the composer to arrange exact pitches in a particular register. Especially in a canonic sense, fixed contour acts as one of the most essential musical elements to help guide the listen through the composition. Without this contour relationship, the listener could become overwhelmed.

Rhythmic Models

The rhythmic aspect of *Primus* not only fulfills the expectations of a traditional mensuration canon, but adds an interesting element proper to the 20th century music composition. By retrograding the rhythm of the *comes*, Dallapiccola modifies the harmonic structure of the movement and redefines the mensuration canon. It can be immediately recognized that the rhythmic value of the *comes* is a diminution of the *dux* by ¾ and is also retrograded. Example 3.7 shows the rhythmic model used (note that the *comes* appears in the actual music two half-note beats later than shown here due to the time interval of the canon).

Example 3.7 - Rhythmic reduction of the first two voices where X is the model rhythm.



Note that the rhythm in both the *dux* and the *comes* retrogrades half way through each of the tone rows (due to the diminution of the *comes*, rests must be added to the rhythmic model to compensate for the rhythmic reduction. Without the addition of rests the *dux*, having smaller rhythmic values, would eventually be ahead of the *comes*).

Trichordal and Hexchordal Partioning

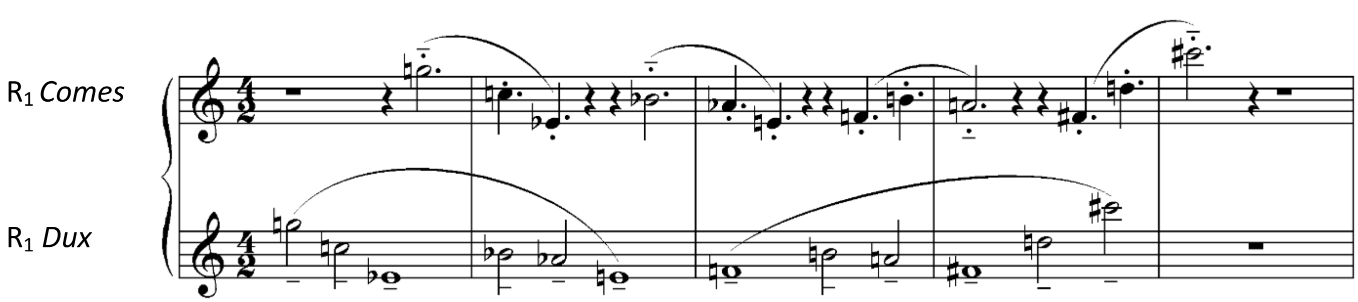
Through the rhythmic model shown in example 3.7, Dallapiccola has partitioned the tone-row into four segmental sections. Firstly, as observed in example 3.7, one can see that Dallapiccola divides the tone row into trichordal cells through rhythmical units. Each unit, in both the *dux* and the *comes*, appears twice, then appears again in its retrograde. Each statement of the canon suggests successive partitioning of each tone row into hexachordal cells (see example 3.8). Dallapiccola has partitioned the tone row evenly into two hexachords, through temporal means and through his slur markings. In the first statement of the *dux*, Dallapiccola partitions the tone row into two hexachordal cells through phrasing and by symmetrically retrograding the rhythm. When the *comes* appears, the tone-row is partitioned into trichordal cells through the use of quarter rests. Moreover, the slur markings in the *comes* also evenly partition the tone-row into four trichordal cells, and the articulation markings on the *comes* further reinforce Dallapiccola's trichordal partitioning, by placing *tenuto* markings on the first note of every trichordal cell, or the last in the retrograde. Example 3.8 is a re-score of the opening measures of *Primus.*

Example 3.8 – Tone row P_e as it appear in *Primus*, measures 1-5.

In Thomas DeLio's analysis of Dallapiccola's second song from *The Goethe Lieder*, similar observations are made[15]. Within each of the two sections of the song, Dallapiccola strategically places long and short rests which separate the two tone rows and partitions each of the tone rows into hexachords. The long rests separate hexachords and the short rests to further partition a tone row into trichords.

Dallapiccola maintains this type of partitioning throughout the movement. When the next tone row is introduced, R_1, his slur and articulation markings remain. However, the B section becomes increasingly more complex, particularly in the rhythmic aspect. Example 3.9 is a re-scoring of the second half of the A section and example 3.10 re-scores the B section.

Example 3.9 – Tone row R_1 as it appears in *Primus*, measures 5-9.



[15] Thomas DeLio, "A Proliferation of Canons: Luigi Dallapiccola's "Goethe Lieder No. 2"" *Music Theory Spectrum* Vol. 23, No. 1 (2001): 1.

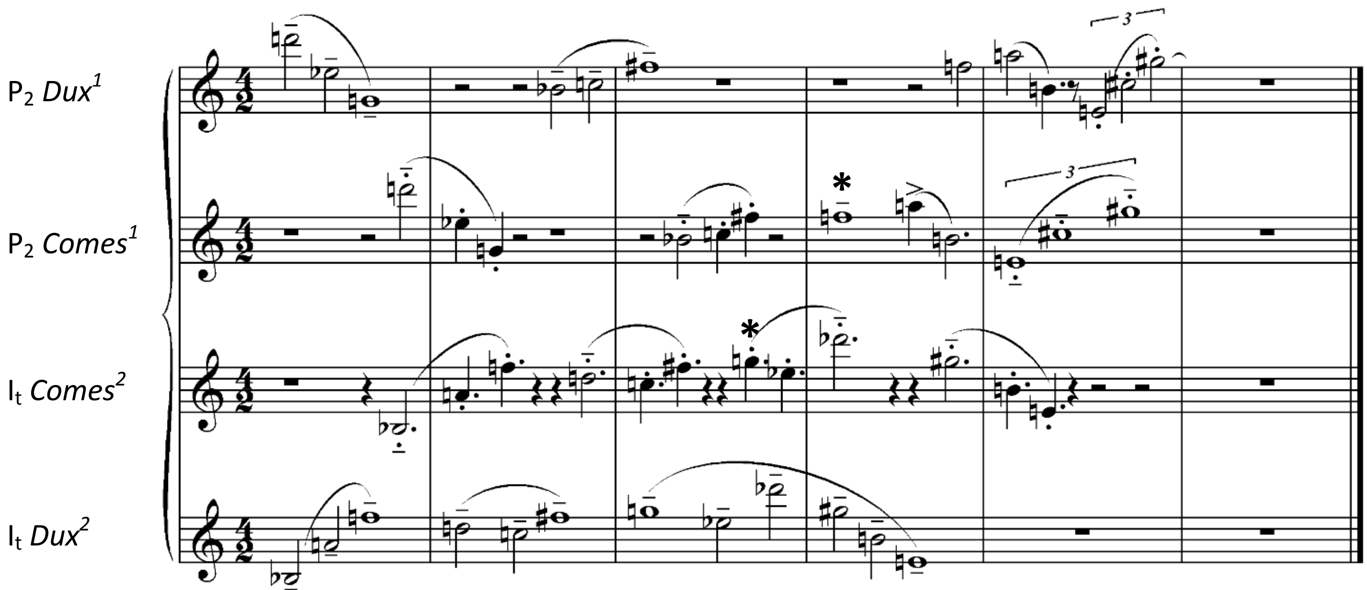
The B section becomes more complex with the addition of another voice. This new voice (labeled as *dux*[2]) introduces tone row R_4 and creates a canonic relationship with the *comes* while the original *dux* (labeled as *dux*[1]) maintains the canonic relationship at the unison with the *comes*. Notice that Dallapiccola also maintains trichordal partitioning through the use of phrase and articulation markings. Though *dux*[2] is, in comparison, an irregular rhythmic addition to the movement, it is still clearly broken down into trichordal cells. Also, for the first time, the *dux*[1] is partially divided into two trichordal cells and one hexachordal cell.

Example 3.10 – The B section of *Primus*.

Though section C, see example 3.11, becomes more erratic rhythmically, Dallapiccola continuously maintains trichordal and hexachordal partitioning. With a total of four voices, a double canon is at play here. Once each *dux* plays through half of the tone row, the *comes* take over the role of the *dux*, and the *dux* then becomes the *comes*. In Example 3.11, this exchange in roles is marked by an asterisk. Canonic relationships will proliferate between each of the four voices in a double canon. Since Dallapiccola has chosen to close the movement with tone rows

P_2 and I_t, which are literal complements of one another, it makes the closing measures of *Primus* quite rich in terms of pitch content.

Example 3.11 – Section C of *Primus*. Note that *dux*^{1} and *comes*^{1} are notated on one staff in the original score.



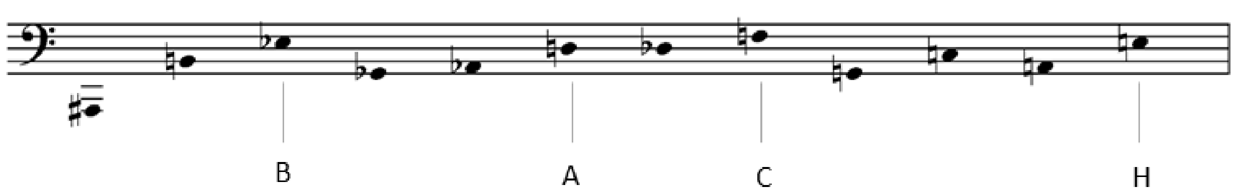
Cross Partitioning

As discussed in chapter two, cross partitioning serves as a motivic and referential tool in *Primus*. Motivically, Dallapiccola uses cross partitioning to isolate the BACH motive. First presented in the opening movement of the *Quaderno*, Dallapiccola uses cross partitioning and registral placement to isolate the motive. Because the BACH motive does not appear in sequence in his twelve-note series, he has to extract it through the use of an "uneven" cross partition.

Before discussing the variants and transformations that the BACH motive undergoes, a brief introduction of the BACH motive in its original form is necessary. The first five measures of

Simbolo are shown in example 3.12; below I have included the ordering of the series. In order to make the BACH motive audible, it is placed in the top register and is metrically accented as the first beat of each measure. This is an example of an uneven cross partition.

Example 3.12 – Opening measure of *Simbolo.*



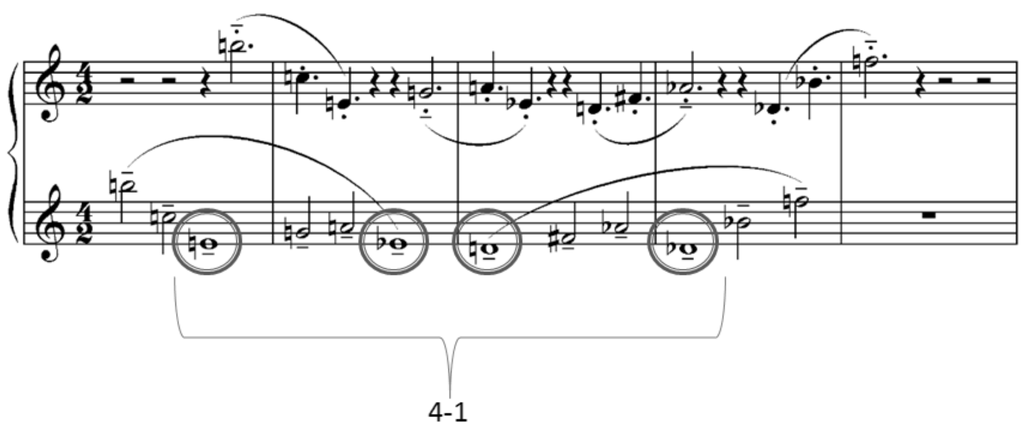
Alegant credits Dallapiccola's non-segmental extraction of the BACH motive to the influence of Schoenberg's *Variations for Orchestra Op.31.* Similarly to *Simbolo*, Schoenberg differentiates the BACH motive registrally and timbrally.[16]

Through the use of the rhythmic units, Dallapiccola has partitioned (not to be confused with cross partitioned) the tone-row into four discrete trichords, these also marked by his phrasing. One way in which a composer can isolate notes is by putting them in a particular register as seen in *Simbolo*. In example 3.12, Dallapiccola focuses on the highest notes. The chromaticism of the BACH motive is isolated in the highest register. Since the BACH motive

[16] Brian Alegant, *The Twelve-Tone Music of Luigi Dallapiccola* (Rochester: University of Rochester Press, 2010), 37.

serves as thematic material throughout the collection, it is expected to appear in its original form, as well as variant forms. The BACH tetrachord is further accentuated because it is given longer note values in the statement. In example 3.13, Dallapiccola isolates the BACH tetrachord in the lowest register, even though it is unordered and transposed. Note that if this is true in the statement of the canon, it will also true in the following voice if the canon is in unison.

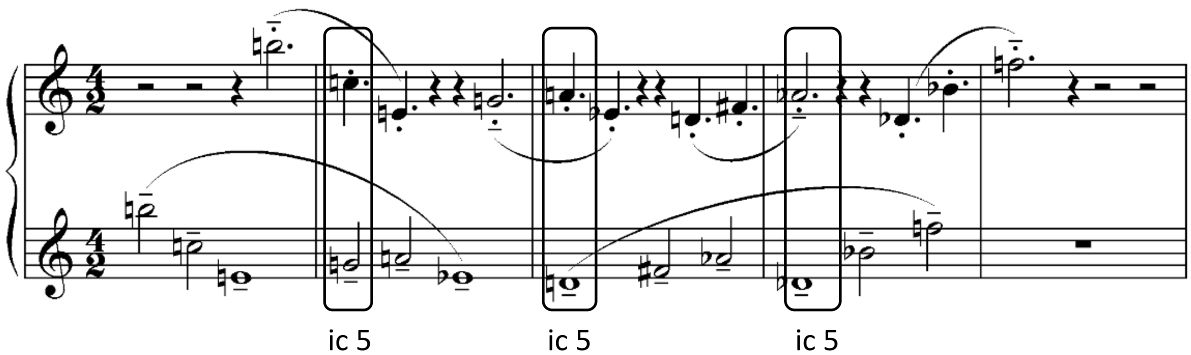
Example 3.13 – The BACH motive in *Primus.*



Due to a consistent rhythmic arrangement in *Primus*, certain order positions in the voices will always be in alignment. Section A seems like a fairly straightforward mensuration canon with the added bonus of a retrograded rhythm. However, every adjustment made to the row will inevitably cause a change in another component of the movement and, therefore, affect how tone rows interact with one another.

Section A is occupied by only two complementary tone rows; P_e and R_1. The first half of section A contains P_e in the *comes* which is separated from the *dux* at two half and a quarter beats. This rhythmic separation will inevitably articulate an interval class 5 (ic 5) between the two voices.

Example 3.14 - The first beat of each measure is outlined creating ic 5 within the P_e tone row.



This type of cross partitioning does not fall under any of the regular patterns defined by Alegant. Instead, we find that Dallapiccola chooses to highlight the saturation of ic 5 in his series.

Table 3.2 – Cross partitioning in *Primus*, tone-row P_e.

Order Position	0	1	2	3	4	5	6	7	8	9	10	11
P_e	e	**0**	4	**7**	**9**	3	**2**	6	**8**	**1**	t	5

ic 5 ic 5 ic 5

In the second half of section A, Dallapiccola employs tone row R_1. The same order positions are superimposed, but since R_1 is extracted from a different row form different interval classes are accented. The slight overlap between the end of P_e and the beginning of R_1 creates an ic 2, making the total of accented interval classes to two ic 2 and two ic 3 for the second half of section A.

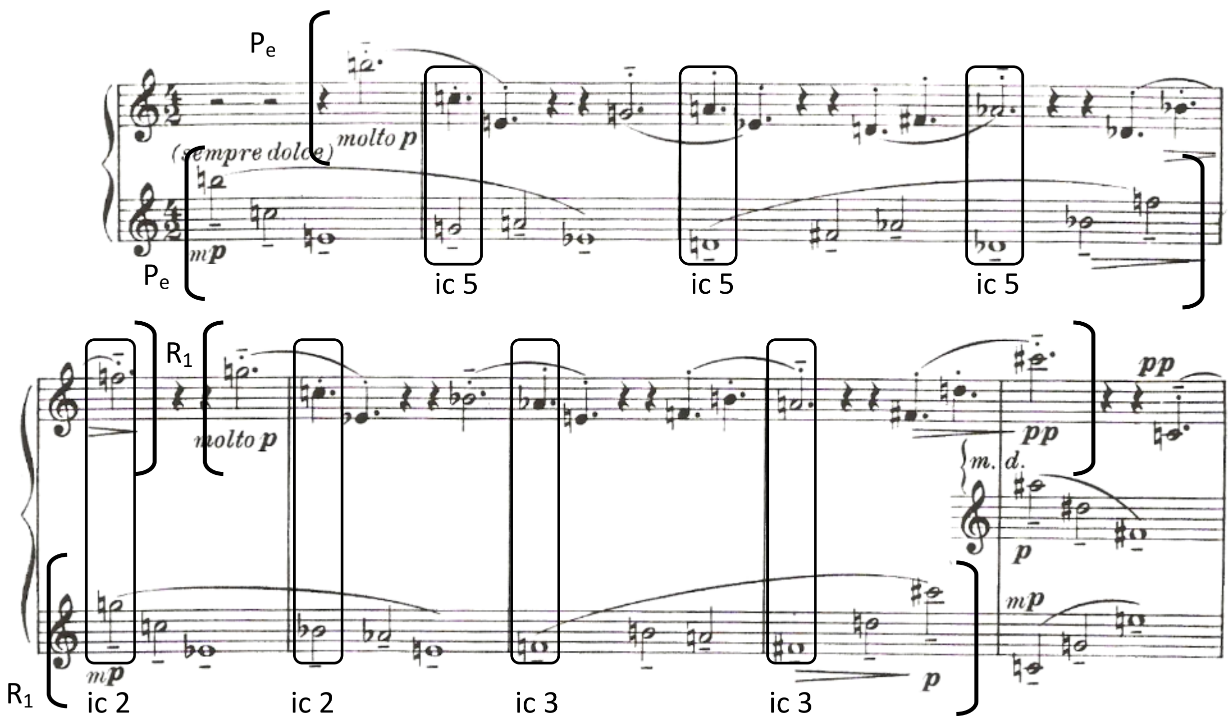
Table 3.3 – Cross partitioning in *Primus*, tone-row R_1.

Order Position	0	1	2	3	4	5	6	7	8	9	10	11
R_1	7	**0**	3	**t**	**8**	4	**5**	e	**9**	**6**	2	1

$\underbrace{\textbf{0}\ 3\ \textbf{t}}_{\text{ic 2}}$ $\underbrace{\textbf{8}\ 4\ \textbf{5}}_{\text{ic 3}}$ $\underbrace{\textbf{9}\ \textbf{6}}_{\text{ic 3}}$

This particular organization creates a sense of metric accenting through interval class. Example 3.15 outlines the first beat of each measure in section A.

Example 3.15 – The A section of *Primus.*



In section B, a new voice enters with tone row R_4 accompanied by the outer voices with IR_6. The newest voice (tone row R_4) interacts with the upper voice as a mirror canon and the lower voice as a mirror-mensuration canon. The outer voices continue in the same manner by pairing the same order positions as was the case for section A.

Figure 3.16 – The B section of *Primus.*

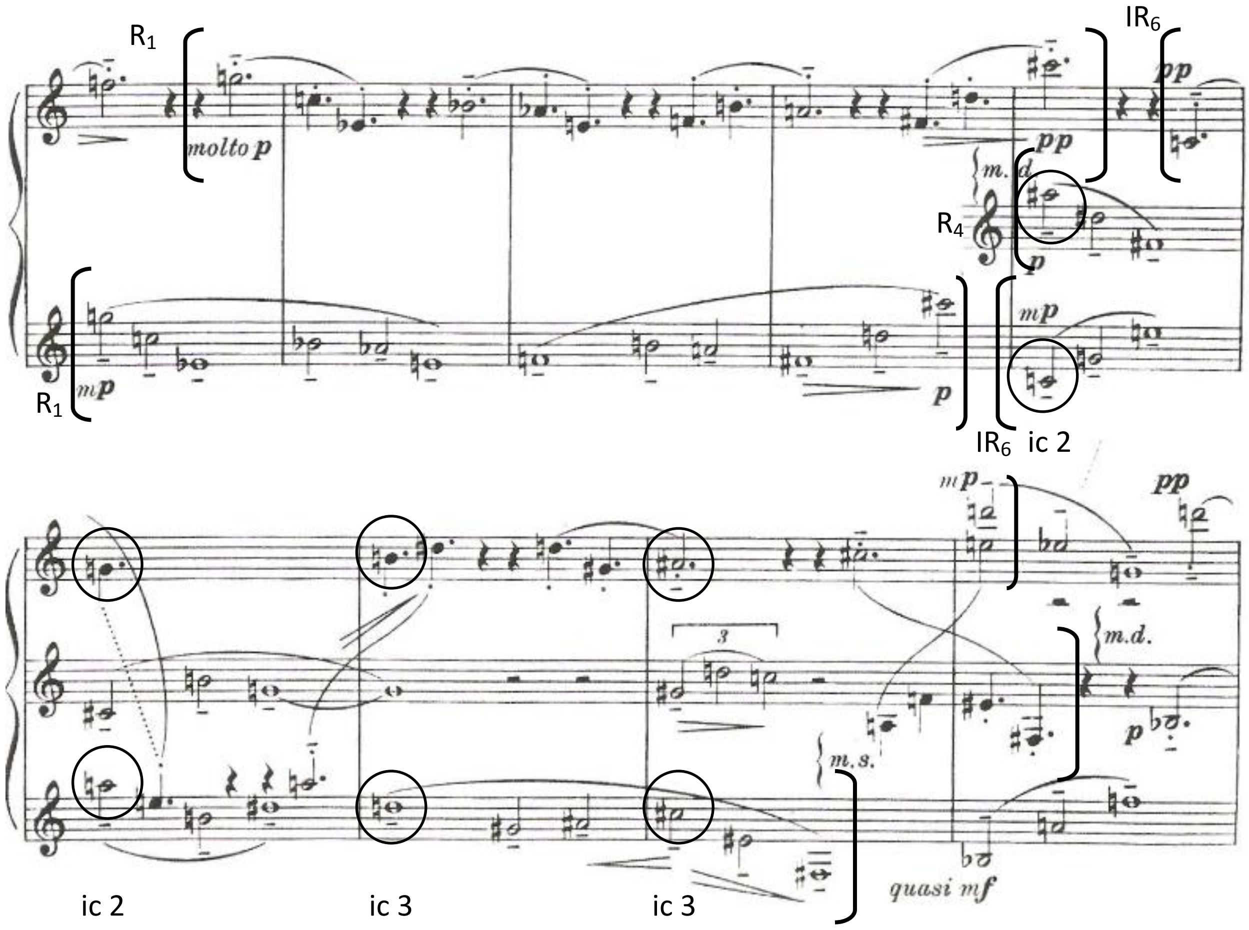


Table 3.4 – Cross partitioning in *Primus*¸tone-row IR$_6$.

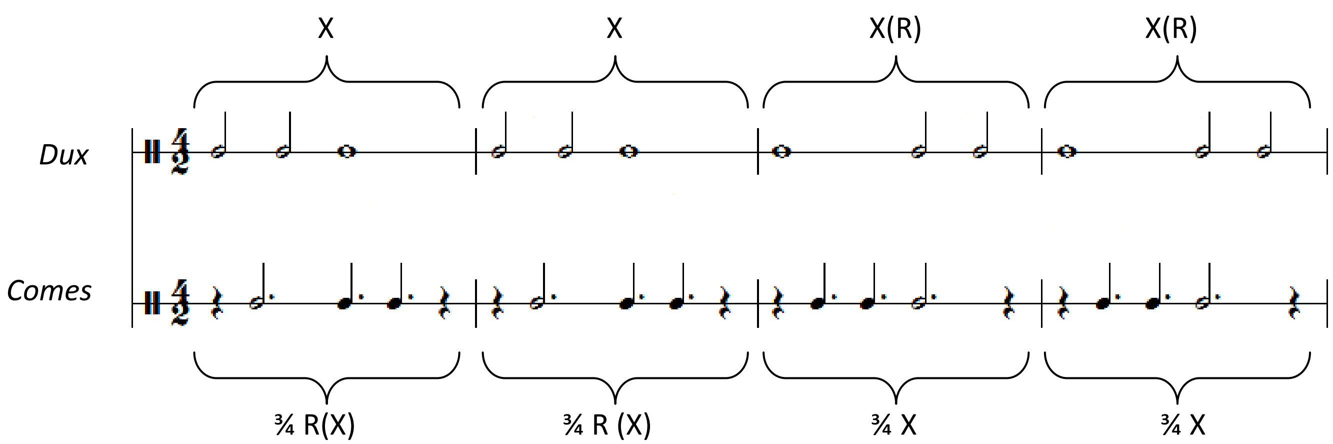
Order Position	0	1	2	3	4	5	6	7	8	9	10	11
IR$_6$	0	**7**	4	**9**	**e**	3	**2**	8	**t**	**1**	5	6

ic 2 ic 3 ic 3

Note that the elision from the previous tone row in the upper voice does not create an ic 2 as it did in the second half of section A. However, with the entry of the new voice on R$_4$, an ic 2 is still created allowing for a full repetition of the cycle.

The addition of R_4 promptly complicates things. Now serving as a double canon, section B offers a little more rhythmic complexity than section A. Adding another voice to, but not another staff, section C is the final section of the short movement employing tone rows P_2 in the upper voices and I_t in the lower voices. The upper voices switch roles (*dux* to *comes* and *comes* to *dux*) halfway through the row. The lower two voices which are in canon at the unison are presented in the same manner as the canon at the beginning of section A, except the voices have changed direction and the retrograde pattern is altered. Figure 3.17 shows a rhythmic reduction of the two lower voices from section C. The rhythmic values appear almost exactly as they did in section A, but a closer look reveals that the last group in both voices does not appear as retrograded. Note that the *comes* appears two half-note beats later than shown here due to the time interval of the canon.

Example 3.17 – Rhythmic reduction of the first two voices where X is the model rhythm.



Polarity

It is appropriate to follow the subject of *Cross Partitioning* with *Polarity* as Dallapiccola exploits the flexibility of the cross partitioning method to achieve a sense weight. In the previous section I observed that Dallapiccola accentuates interval class 5, 3, and 2 throughout *Primus*. The following data supports this emphasis on these interval classes. I have recorded the number of times and the interval quality that occurs on the strong beat of each measure. Table 3.5 is a collection of the harmonic intervals which occur on each strong beat in *Primus*. I suggest that the frequency of a particular interval class and the timing in which it arrives, gives a significant amount of importance to the interval class.

Table 3.5 – Harmonic intervals articulated on the strong beat in *Primus*.

Interval Class / **Section**	**1**	**2**	**3**	**4**	**5**	**6**
A Section	0	2	2	0	3	0
B Section	1	3	3	2	2	1
C Section	3	2	3	0	6	4
TOTALS	4	<u>7</u>	<u>8</u>	2	<u>11</u>	5

From the data collected, it is clear there is an emphasis put on interval class 5, and secondly on interval classes 3 and 2. I should point out that an analysis based entirely on the frequency of an interval class would be a rather superficial one. I would like my data to demonstrate how the interval classes are revealed and where Dallapiccola chooses to reveal them. In the opening measure of *Primus* we are first presented with three occurrences of interval class 5 which establishes the *polarity* of the pieces, the same way in which the harmony in the beginning of a tonal piece establishes the tonic[17]. Following interval class 5, Dallapiccola presents short fragmentations of interval-classes 2 and 3. I interpret these fragmentations as a transition to the B section. The B section, as observed in table 3.5, is intervalically sporadic, including every interval class. However, the B section provides the listeners ears with the highest concentration of interval class 3 and 2. The C section becomes increasingly more complicated with the addition of a fourth voice. Notice that there are six occurrences of interval class 5 in a section where there are only five strong beats to record. The extra interval class 5 occurs in the last measure of the piece due to the addition of the extra voice.

I suggest that the polarity in *Primus* primarily, and its central focus, then is interval class 5. Secondarily, interval classes 2 and 3 act as transitional areas throughout the movement. Lastly, the abundance of interval class 5 in sections A and C supports the idea of impressing a particular interval upon the listener's memory in order to emphasize a central aspect of a piece of music. The impression of interval class 5 at the beginning and the end of the movement pulls the listener into the looping effect of a perpetual canon brining their ears from the end of the piece back to the beginning. The ABA form of the movement is thus determined by the stable and unstable sections based on polarity. These sections are invariantly related through segmental and non-segmental parts of the tone row. The invariant relationships within the tone rows creates a sense of unity for the movement through these overlapping features of the series. Other aspects of the tone-row construction such as the contour preservation also demonstrates the subtle relationships between tone rows. The contour features of this

[17] *Polarity* is the attraction between two pitch classes (Straus refers to these as *poles*) or the attraction of a particular pitch-class interval(s) used to establish centricity in a piece of music.

movement is the most helpful guide to the listener. Ordered intervallic relationships may be difficult to aurally perceive, therefore, the listener may need to rely on broader aspects of the music such as the general contours of the series to guide them through the composition.

N. 5 – Contrapunctus Secundus (Canon Contrario Motu)

Tone-Row Realization

Contrapunctus Secundus (hereafter referred to as *Secundus*) is the second contrapuntal work to appear in the *Quaderno* and is separated from *Primus* by one movement. *Secundus* offers a denser presentation of the tone rows in comparison to *Primus*. Its lively tempo and quick rhythmic gestures differ greatly to that of the smooth motion of *Primus*. This canonic work is an inversional canon separated by an eighth-note beat and is divided into an equal binary form, each section made of four measures each which are further divided into 2 sub-sections. Section A (mm.1-4) presents a tone row from each type of row form: P_7, I_5, R_3, and IR_9. The second half, section B (mm. 5-8), similarly uses each type of row form: P_t, I_8, R_6, and IR_0. Table 3.6 shows the tone rows used in *Secundus*.

The relationships between the tone rows (transpositionally/inversionally) are expressed in table 3.6. Dallapiccola is very systematic about his tone-row selection. The types of operations Dallapiccola applies to the tone rows in the first half of the piece stay the same, with the exception of the relationship between voice 1 and 2 in each section. In the first section, Dallapiccola uses literal complements, expressed as red arrows in the diagram in table 3.6. In the section half of the piece, voice 1 and 2 are related by T_6I. This change causes the closing measure of each section to generate a unique feature of this piece, a quasi-sense of cadence.

Table 3.6 - Transpositional relationships between tone rows in *Secundus*.

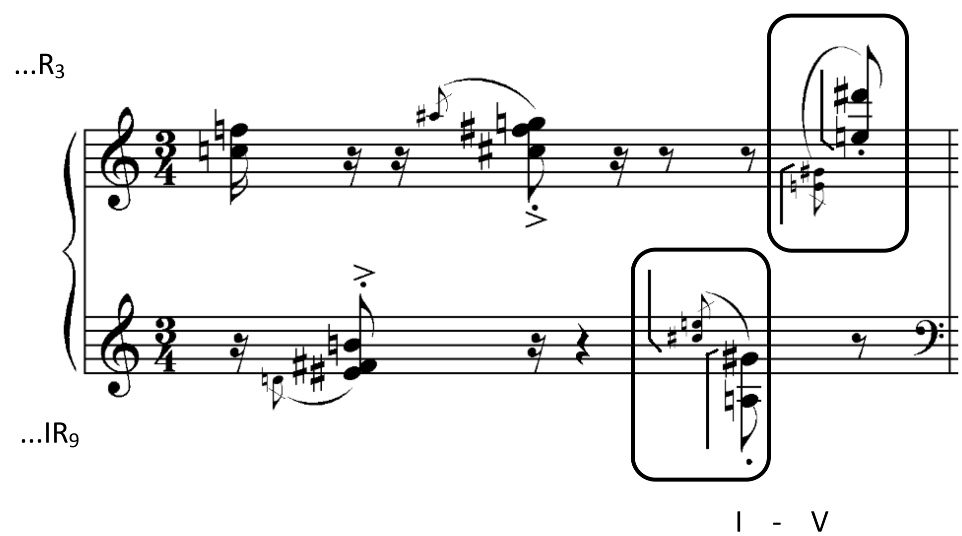
	m.1-3	**m. 3-4**	**m. 5-7**	**m. 7-8**
	Section A		**Section B**	
Voice 1	7 8 0 3 5 e t 2 4 9 6 1 P_7	9 2 5 0 t 6 7 1 e 8 4 3 R_3	t e 3 6 8 2 1 5 7 0 9 4 P_t	0 5 8 3 1 9 t 4 2 e 7 6 R_6
Voice 2	5 4 0 9 7 1 2 t 8 3 6 e I_5	3 t 7 0 2 6 5 e 1 4 8 9 RI_9	8 7 3 0 t 4 5 1 e 6 9 2 I_8	6 1 t 3 5 9 8 2 4 7 e 0 RI_0

Relationship
Literal Complement
T_6I
Retrograde T_8
Retrograde T_4
$T_{4(8)}$
$T_{8(4)}$
T_3

Cadences

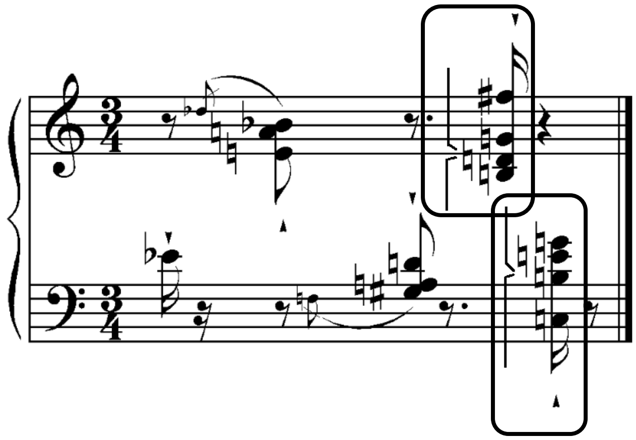
Dallapiccola's tone-row selection may have been based on generating what we call a half cadence at the end of the A section, and "perfect cadence" at the end of the B section. The first tetrachord in the series is 4-20 (0158), which can also be referred to, tonally, as a major seventh chord. In the A section, Dallapiccola chooses tone rows R_3 and IR_9 to close the section, which will generate two tetrachords, one played immediately after the other. With the first tetrachord presented as <1 4 8 9> or C# E G# A and the second tetrachord following as <e 8 4 3> or B G# E D#, Dallapiccola creates a quasi-sense of a half cadence in A major. Example 3.18 outlines these tetrachords.

Example 3.18 - Closing measure of the A section from *Secundus*.



In the second half of the piece another quasi cadence in C major occurs at the closing measure (see example 3.19). The B section ends with tone rows R_6 and IR_0. Once again, both tone rows will end in tetrachord 4-20. The first tetrachord presented is <2 6 7 e> and the second tetrachord as <0 e 4 7>, another quasi cadence and a quasi-perfect cadence in C major.

Example 3.19 - The last measure of *Secundus*.

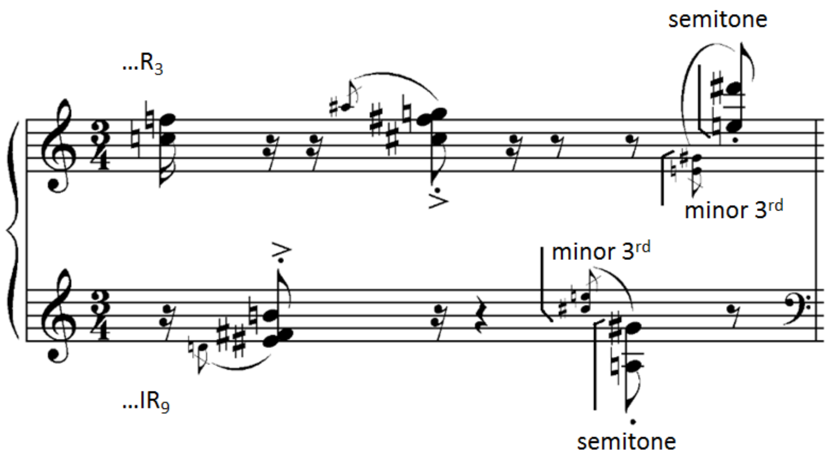


The quasi half cadence at the end of the A section and the quasi perfect cadence at the end of the B section strongly support a binary form. Dallapiccola was also fond of using cross partitioning to punctuate sections within a piece of music.

Cross Partitioning

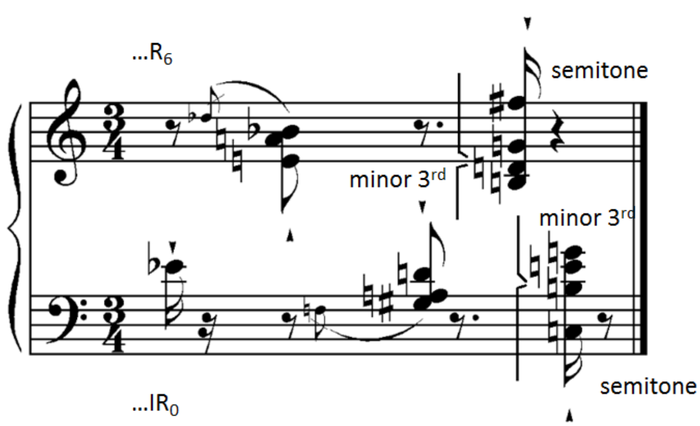
Used to punctuate the sections of the movement, variations of the BACH motive appears at the end of each section of the binary form. Dallapiccola often used the cross partitioning to mark the beginning and ending of a section or entire piece of music. Instead of presenting the BACH motive linearly like in *Primus*, Dallapiccola accentuates the intervallic content of the motive. Aurally, the BACH can be perceived as two semitones separated by a minor third. These two intervallic features are isolated in *Secundus.* The end of the A section in measure 4 closes with tone rows R_3 and IR_9 and brings out the semitone and minor third intervals of the BACH motive (see example 3.20).

Example 3.20 – Closing measure of the A section from *Secundus.*



Similar interval accentuations occurs in the B section which closes with tone rows R_6 and IR_0. The same row presentation is given, but is altered rhythmically and is played as one sonority. Note that a separation of dyads is still at play, marked with brackets, Dallapiccola indicates to the performer that the top two notes (F-sharp and G) are to be played with the right hand, and the bottom two notes (B and D) are to be played with the left hand.

Example 3.21 – The last measure of *Secundus.*



Aurally, the listener would not be able to distinguish which notes are being played by which hand of the performer. However, in 1954 when Dallapiccola orchestrated the *Quaderno*, he separated the semitone and the minor 3rd by placing them in different instruments, making this occurrence of the BACH motive more easily perceptible. According to Alegant, using cross partitions to punctuate sections of a piece is typical of Dallapiccola.[18] This is not the last time which Dallapiccola uses this variation of motivic presentation. Dallapiccola's tone-row selection is not only crucial to motivic variation, but also determine this movements sense of polarity.

Polarity

Centricity can be established in many different ways – by stating pitches longer, louder, more often, higher, and/or lower. In *Primus* I observed above that a centric interval class was established early in the movement through repetition and time of arrival. According to Straus, centricity could be based on inversional symmetry, a pitch (and its counterpart) in which all pitches are centered around.[19] The inversional axis of a piece can be determined by finding the index number (sum). To find the index number one simply has to add the corresponding elements of the tone-rows to find out if they are inversionally related (see tables 3.7 and 3.8). When the corresponding elements are added, they should all be equal, therefore, will be related by inversion. The A section of *Secundus* employs a set of tone-rows that are literal complements. This means that they are related by the operation T_0I. In other words, the A section will have an inversional axis of C/F#. In figure 3.2, the line connecting pitch classes on the clock face show which pitch classes will map onto each other.

[18] Brian Alegant, *The Twelve-Tone Music of Luigi Dallapiccola* (Rochester: University of Rochester Press 2010), 21.
[19] Straus, Joseph. *Introduction to Post-Tonal Theory* (New Jersey: Prentice Hall, 2005), 133.

Table 3.7 – Index number for tone-rows P_7 and I_5.

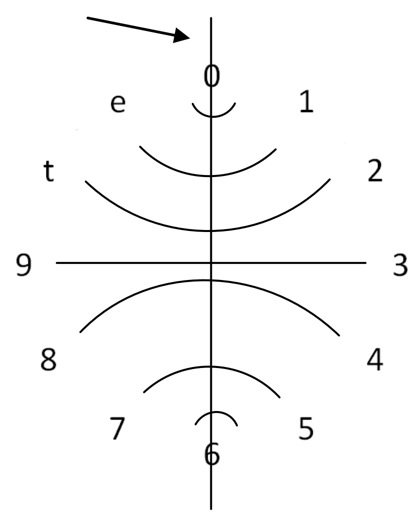
P_7	7 8 0 3 5 e t 2 4 9 6 1
I_5	5 4 0 9 7 1 2 t 8 3 6 e
Index Number (sum)	0 0 0 0 0 0 0 0 0 0 0 0

Table 3.8 – Index number for tone-rows R_3 and IR_9.

R_3	9 2 5 0 t 6 7 1 e 8 4 3
IR_9	3 t 7 0 2 6 5 e 1 4 8 9
Index Number (sum)	0 0 0 0 0 0 0 0 0 0 0 0

Figure 3.2 – Axis of symmetry for the A section of *Secundus.*

Axis of Symmetry 0 - 6



The B section employs tone rows which are also related by inversion, but are related through a different index number (see table 3.9 and 3.10); therefore, the B section will have a different inversional axis. Each of the tone-rows which are in canon with one another is related by T_6I.

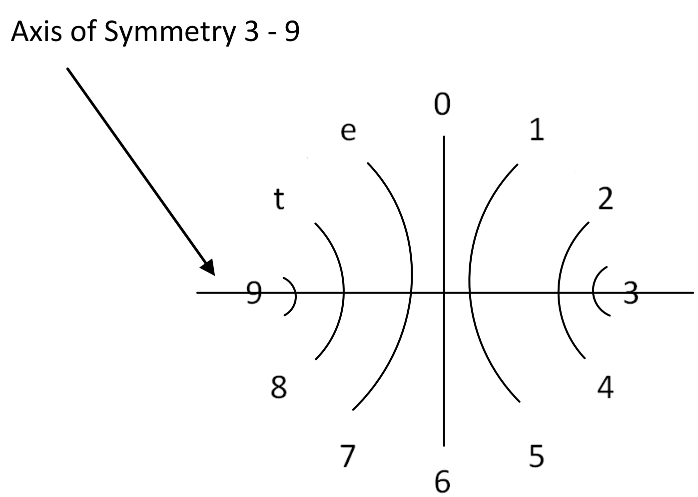
Table 3.9 – Index number for tone-rows P_t and I_8.

P_t	t e 3 6 8 2 1 5 7 0 9 4
I_8	8 7 3 0 t 4 5 1 e 6 9 2
Index Number (sum)	6 6 6 6 6 6 6 6 6 6 6 6

Table 3.10 – Index number for tone-rows R_6 and IR_0.

R_6	0 5 8 3 1 9 t 4 2 e 7 6
IR_0	6 1 t 3 5 9 8 2 4 7 e 0
Index Number (sum)	6 6 6 6 6 6 6 6 6 6 6 6

Figure 3.3 – Axis of Symmetry for the B section of *Secundus*.



Because the inversional axes are odd numbers, the axis of symmetry will pass through two pitch classes (see figure 3.3). Dallapiccola deliberately places a Schoenbergian accent (meaning he does not accentuate the note) on the first C and the first E-flat in both sections, for fear of revealing too much of the movement's structure at once. These two axes serve as centric areas for each section of the piece, suggesting a binary formal structure.

Contour

As in *Primus*, the registral presentations of the tone rows in *Secundus* are also related by contour. Although the contours are not identical, tone-row forms reflect similar overall contours. Figure 3.22 shows the tone rows in *Secundus*.

Figure 3.22 - Tone rows P_7, I_5, R_3, IR_9, P_t, I_8, R_6, and IR_0 as they appear in *Secundus*.

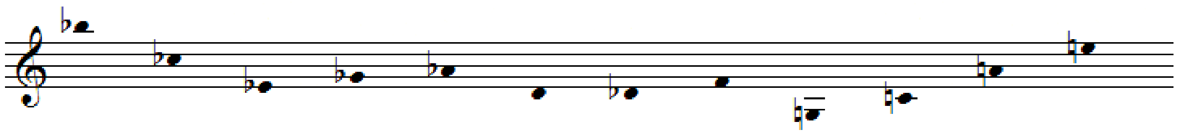
P_7

I_5

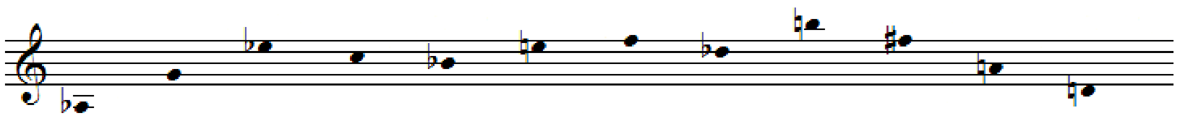
R_3

IR_9

P_t

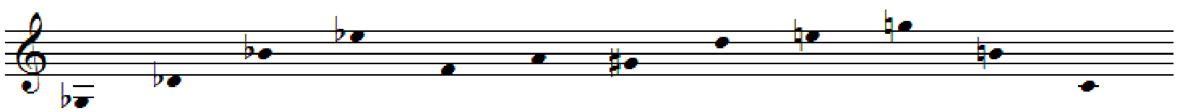


I_8



R_6

IR_0



Dallapiccola maintains much the same registral choices as he did in *Primus* which will also generate the exact same contour relationships. The melodic and inversional nature of this canon uses these contour similarities to guide the listener along the *dux* and *comes* statements.

N. 7 – Andantino Amoroso e Contrapunctus Tertius (Canon Cancrizans)

Tone-Row Realization

Contrapunctus Tertius (hereafter referred to as *Tertius*) is the last and the densest of the contrapuntal movements in the *Quaderno*. The four tone rows exploited in this movement are stated at the top of the published score in red ink: P_t, R_5, I_3, and IR_e. Table 3.11 expresses the tone-row presentation in *Tertius*. Each tone row is also transformed into its retrograde form, in order to establish the crab canon.

Table 3.11 – Tone row realization of *Tertius*.

Voice	Section A mm. 1-8	Section B mm. 8-12	Section C mm. 13-17
1	$P_t \rightarrow R_5 \rightarrow I_3 \rightarrow IR_e$	$P_t \rightarrow R_5$	$I_3 \rightarrow IR_e$
2		$IR_3 \rightarrow I_e$	$R_t \rightarrow P_5$

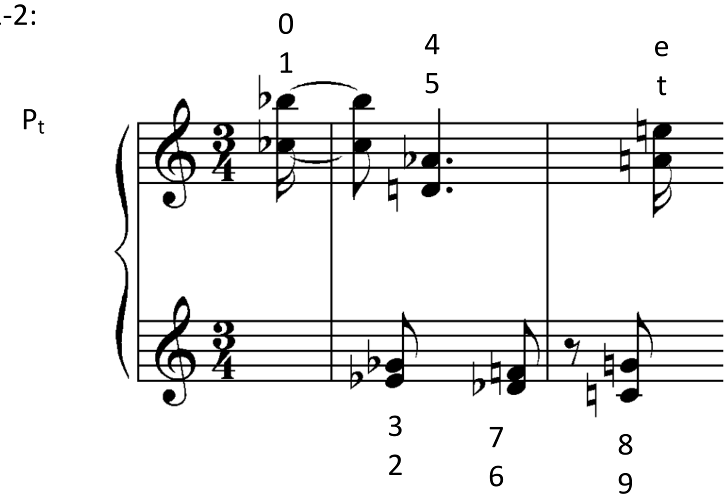
After an initial statement of all four tone rows, the rows are repeated in the top voice while the retrograded forms of each tone row is played out in the lower voice, beginning with the last two tone rows and finishing with the first two.

Cross Partitioning

The dyadic nature of this movement results from Dallapiccola's use of the cross partitioning device. Example 3.23 shows how the first tone row, P_t, is presented.

Example 3.23 – Presentation of P_t in *Tertius* and the order numbers of the series.

Measures 1-2:



While cross partitioning was used irregularly in *Primus* between two different voices, in *Tertius* Dallapiccola uses a 2^6 cross partition in its regular configuration in a single voice for tone rows P_t and R_5. Figure 3.27 and table 3.12 show how a 2^6 cross-partitioning layout is applied to P_t.

Figure 3.27 – The use of a regular 2^6 cross-partition for tone row P_t.

Linear Presentation of P_t:

Order Number	0	1	2	3	4	5	6	7	8	9	t	e
P_t	t	e	3	6	8	2	1	5	7	0	9	4

Table 3.12 – Presentation of P_t in *Tertius*.

	Pitches	Order Numbers	Pattern (slot machine)
P_t	t 6 8 5 7 4 e 3 2 1 0 9	0 3 4 7 8 e 1 2 5 6 9 t	↓ ↑ ↓ ↑ ↓ ↑

The purpose of the "slot machine" transformation, as Alegant refers to it as, is to alter the horizontal elements of the collection and preserve the elements of the vertical dimensions. When tone row R_t occurs at the end of the movement, Dallapiccola arranges the pitches so that the same "slot machine" pattern is preserved. Example 3.24 and tables 3.13 and 3.14 show the presentation of R_t.

Example 3.24 – The use of a regular 2^6 cross-partition for tone row R_t.

Measures 15-17:

Table 3.13 – Linear Presentation of R_t.

Order Number	0	1	2	3	4	5	6	7	8	9	t	e
R_t	4	9	0	7	5	1	2	8	6	3	e	t

Table 3.14 – Presentation of R_t in *Tertius*.

	Pitches	Order Numbers	Pattern (slot machine)
R_t	4 7 5 8 6 t 9 0 1 2 3 e	0 3 4 7 8 e 1 2 5 6 9 t	↓ ↑ ↓ ↑ ↓ ↑

This particular type of cross-partitioning occurs for tone rows P_t and R_5 along with each of their retrogrades. Table 3.15 shows a summary of the tone rows that are in a regular 2^6 cross-partitioning pattern discussed thus far. Notice that the order numbers 2 and 3 are swapped in tone row R_5 (shaded arrows), but the vertical sonority is still preserved.

Table 3.15 – Cross-partitions in *Tertius*, tone-rows P_t and R_t.

	Pitches	Order Numbers	Pattern (slot machine)	Hexachord
P_t	t 6 8 5 7 4 e 3 2 1 0 9	0 3 4 7 8 e 1 2 5 6 9 t	↓ ↑ ↓ ↑ ↓ ↑	6-2 (012346)
	Pitches	Order Numbers	Pattern (slot machine)	Hexachord
R_t	4 7 5 8 6 t 9 0 1 2 3 e	0 3 4 7 8 e 1 2 5 6 9 t	↓ ↑ ↓ ↑ ↓ ↑	6-2 (012346)

Table 3.16 – Cross partitions in *Tertius,* tone-rows R_5 and P_5.

	Pitches	Order Numbers	Pattern (slot machine)	Hexachord
R_5	e 7 0 3 1 5 4 2 8 9 t 6	0 2 4 7 8 e 1 3 5 6 9 t	↓ ↓ ↓ ↑ ↓ ↑	6-22 (012468)
	Pitches	Order Numbers	Pattern (slot machine)	Hexachord
P_5	4 7 5 8 6 t 9 0 1 2 3 e	0 3 4 7 8 e 1 2 5 6 9 t	↓ ↑ ↓ ↑ ↓ ↑	6-2 (012346)

Due to the symmetrical construction of the series with two 6-31 hexachords, the hexachords generated under the same “slot machine” pattern will always result in a 6-2 hexachord. However, because the order position in R_5 is swapped, there will be two new hexachords generated, 6-22. The presentations of the other two tone rows in *Tertius* do not follow a regular cross partition as P_t and R_5.

Tone-rows I_3 and IR_e do not conform to any of the regular configurations (see figure 3.4). Instead of a dyadic arrangement I_3 opens with a single pentachord, followed by a single pitch and three dyads.

Figure 3.4 – Uneven cross partitioning in *Tertius.*

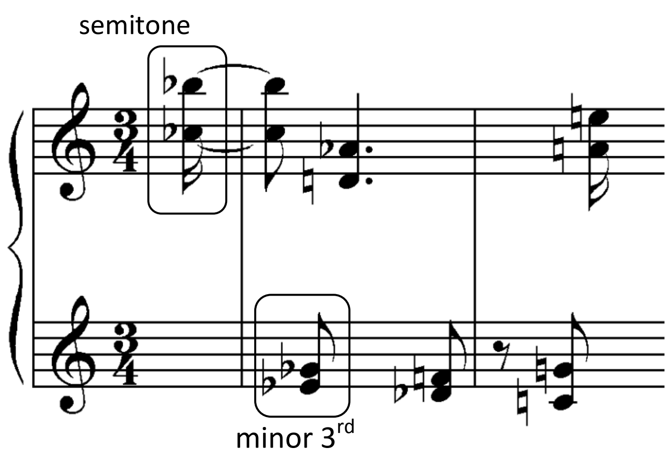
	Pitches					Order Numbers				
I_3	5	e	0	6	4	4	5	6	8	t
	7		8	1	9	3		7	9	e
	2					1				
	t					2				
	3					0				

	Pitches						Order Numbers					
IR_e	5	2	8	7	3	6	0	3	5	6	8	9
	0	9		1		t	1	2		7		t
		4				e		4				e

Though the BACH tetrachord can be described in many different ways, depending on the context, aurally, it can be heard as two semitones in the span of by a minor 3^{rd}. Dallapiccola captures this intervallic content in opening and closing measures as discussed in example 3.20

and 3.21, but example 3.25 from *Tertius* Dallapiccola uses the 2^6 cross partition in an unconventional way. Although this is an even cross partition and conforms perfectly to the 2^6 configuration, the vertical sonorities need addressing. I use the word *unconventional* because the main idea behind the cross partition is to vary the melodic content (via the "slot-machine" transformation), whereas Dallapiccola in this example, is also able to isolate the vertical elements to bring out the BACH motive.

Example 3.25 – Opening measures of *Tertius.*

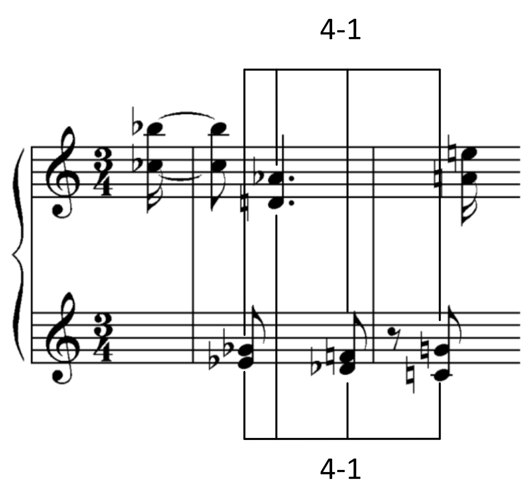


The first two notes, C-flat and B-flat, followed by E-flat and G-flat, summarize the intervallic content of the motive: a semitone followed by a minor 3rd. This is a refreshing presentation of the motive, presented aggregately as opposed to linearly (as it was presented in both *Simbolo* and *Primus*). This example demonstrates how the BACH motive can be presented harmonically, even though pitch classes have changed and therefore changed the tetrachord to a 4-20. Conversely, if this occurs in the opening measures, when tone-row P_t is retrograded at the end of the movement the BACH motive will make another appearance.

Example 3.26 – Tone-row R_t from *Tertius*.

Similarly to *Primus* and *Simbolo*, *Tertius* also includes the BACH motive in a particular register. The lowest notes in P_t and its retrograde form R_t, isolate a 4-1 tetrachord, the same way it was found in *Primus*. At the same time, the 4-1 tetrachord that is in the lowest register is in dyadic pairs. The higher note in each dyad that contains the 4-1 tetrachord, also contains a 4-1 tetrachord in the upper register.

Example 3.27 – The BACH tetrachord in *Tertius*.



Thus far, this is the highest concentration of the BACH motive in this movement. Featured in the opening measures and the closing measures, Dallapiccola uses cross partitioning and the BACH motive to connect the listener's ears back to the beginning of the movement, creating a perpetual canon.

Polarity

The harmonic nature of *Tertius* allows for the impression of a particular interval class to be set into the listener's memory. In *Primus*, I observed that based on the frequency and the imitative moments at which it arrived gave importance to an interval class. I use the same method in *Tertius* to determine its polarity. Each section of *Tertius* is made up of the same tone-rows (or retrograde versions); therefore, each section will have the same number of interval class occurrences. For this set of data, I have decided to collect the interval classes from the dyads and the occasional trichords and pentachords which Dallapiccola partitions throughout the movement. **Table 3.17 –** Harmonic intervals subjected to cross partitioning.

Interval Class / **Section**	**1**	**2**	**3**	**4**	**5**	**6**
P_t	3	0	3	3	6	3
R_5	3	0	3	3	6	3
I_3	3	3	0	6	9	0
IR_e	3	3	0	0	9	3
TOTALS	12	6	6	12	<u>30</u>	9

From the data collected, I found that interval class 5 was the most frequent and had the highest number of occurrences in each of the tone-rows. Since *Tertius* is subjected frequently to cross partition, there seems to be a conscious effort to preserve interval class 5. Recalling the BACH motive, which is presented harmonically in *Tertius*, the emphasis on interval class 1 and 3 is also shown in the data, tied as the second most frequent interval classes.

CHAPTER FOUR

AN ANALYTICAL OFFERING

Dallapiccola's *Quaderno* is compared constantly to J.S. Bach's *Art of Fugue* or *Notebook for Anna Magdalena* without any evidence, however, than its title, the symbolic use of the BACH motive, and the inclusion of contrapuntal movements in the canonic style. In this chapter I would like to offer an analytical approach to the similarities between Dallapiccola's *Quaderno* and Bach's *A Musical Offering* by drawing on characteristics common to both tonality and the twelve-note system, which Dallapiccola so delicately incorporates into his compositions.

N. 3 – Contrapunctus Primus

The two sets of canons in Bach's *Musical Offering* are alluded to in Dallapiccola's *Quaderno* through similar contrapuntal relationships, expressed in a twelve-note context. The first canon to appear in the *Musical Offering* is a perpetual canon at the unison with the King's subject as a *cantus firmus*. Though Dallapiccola's first canon is also a perpetual canon at the unison, this is not the most remarkable similarity. The rhythmic displacement of Bach's first canon allows for the frequent articulation of a perfect fifth, a common intervallic relationship found throughout the work: The introductions of the canonic voices are at a perfect fifth above and a perfect fourth below the King's subject. While many other intervals are used so that the listener does not feel a sense of rest harmonically, the interval of a perfect fifth (also articulated as a perfect fourth) provides the listener with a continuous sense of stability. By impressing the perfect fifth upon the listener's ears through repetition (and without too frequent interruption by any other intervals) the listener is drawn to the perfect fifth and becomes caught up in the cyclic motion of the canon. Through a similarly constant state of harmonic iteration Dallapiccola achieves a perpetual canon using the twelve-tone system. By emphasizing interval-class 5 in the opening measures of the movement and through the abundance of interval-class 5 at the end of the movement he connects the beginning and ending of the movement, but also recalls Bach's use of the imitative perfect fifth.

N. 5 – Contrapunctus Secundus (Canon Contrario Motu)

Many of Bach's canons in contrary motion use the third degree of the scale as a common tone between voices. In *Canon perpetuus (Mirror Canon)* of the *Musical Offering,* the third degree of the scale in C minor (E-flat) is the common tone between the voices. The second pair of voices moves to G minor, but continue to use the third degree of the scale as the common tone (B-flat). Bach also marks the new section off by changing the direction of the voices – instead of the top as the ascending *dux*, it becomes the descending *dux* in the second half of the canon. Similarly, Dallapiccola also uses two different common tones to separate the canon into two sections. As discussed in chapter three, the A section uses C as a common tone, which is the third order position of the series, and also as an axis of symmetry (C and F-sharp). In the B section Dallapiccola chooses tone-rows so that the third order position remains as the common tone between the tone rows, but the axis of symmetry is now changed to E-flat (and A-natural). Therefore, the entire A section is related to the B section through the transposition of a minor third (T_3). Dallapiccola's "modulation" between axes of symmetry is analogous to the way Bach moves from C minor to G minor, canons where both composers use the third note of each system (tonal and twelve-tone) to serve as the common tone for each section accordingly.

N. 7 – Andantino Amoroso e Contrapunctus Tertius (Canon Cancrizans)

Presented as a crab canon, *Tertius* is also a perpetual canon which Dallapiccola indicates in the presentation with repeat signs. Table 4.1 shows the tone rows in *Tertius*: the arrows represent the direction of the tone-row (prime and inversion forms will have forward arrows →, and retrograde and retrograde-inversion forms will have backward arrows ←).

Table 4.1 – Tone-row direction in *Tertius.*

P_t →	R_5 ←	I_3 →	IR_e ←	P_t →	R_5 ←	I_3 →	IR_e ←
				I_e →	IR_3 ←	P_5 →	R_t ←

The first two tone-rows, P_t and R_5, are comparable to the use of the King's subject in Bach's *Musical Offering*. The direction of the material played is the same, and the material similarly moves to the bottom voice in the second half of the canon (see table 4.2). The second two tone rows, I_3 and IR_e, are similar in nature to the counterpoint in Bach's canon and also swap roles to become the top voice in the second half of the canon.

Table 4.2 – Bach's counterpoint direction in *Crab Canon* from *Musical Offering*.

mm. 1-9	mm. 10-18	mm. 19-27	mm. 28-37
King's Subject →	King's subject ←	Counterpoint →	Counterpoint ←
Counterpoint →	Counterpoint ←	King's Subject →	King's Subject ←

These striking similarities do not necessarily suggest that Bach's canons serve as a model of Dallapiccola's canons, but instead demonstrate how much of Dallapiccola's contrapuntal understanding in general is derived from Bach's methods. These comparisons also serve as an example of how Dallapiccola can so creatively manipulate the twelve-tone system to conform to traditional tonal principles.

CHAPTER FIVE

CONCLUSIONS: AVENUES FOR FURTHER EXPLORATION

In this final chapter of this study, I summarize my findings pertaining to polarity for all three movements. I have set out my own method of interpreting the data collected regarding interval class as a result of tone-row partitioning and cross partitioning. I conclude that through partitioning and cross partitioning, a composer can manipulate the order of a twelve-note series and emphasize a particular interval class, pitch class, or pitch class set through a specific set of parameters. for each movement.

Intervallic Stability

In this section I have created pie charts to demonstrate Dallapiccola's use of consonant intervals versus dissonant intervals by finding the interval vector for each movement. Each movement calls for a different set of parameters in determining the interval vector, depending on the technique used to accentuate the interval class(es) or pitch class(es). For *Primus* I record the quality and frequency of the harmonic intervals which are articulated on the first beat of each measure, for *Tertius* I record the quality and frequency of harmonic intervals that result from the 2^6 cross partition and the uneven cross partitions, and for *Secundus* I record the quality and frequency of both the number of melodic skips (where there are linear presentations) and harmonic intervals where there are cross partitions. I alter the parameters for each of the movements since each movement uses different means in establishing polarity. Dallapiccola realized as early as 1925 that a universal analytical model for the twelve-note system was problematic: “From time to time, I tried my hand at analysing atonal works. I went wrong with many of them : with others I was more successful. I noticed that a system of analysis which held good for one work did not hold good for another.”[20] I should also make it clear that my data does not express the time of arrival of the interval classes; the data demonstrates only the amount of consonance and dissonance expressed as a percentage in a given piece of music under the parameters set out for that particular piece.

I have categorized all six interval classes on a scale from most consonant (black) to most dissonant (white). Therefore, the darker the pie chart the more consonant the music is. Interval-classes 5, 4, and 3 are considered consonant while interval-classes 2, 1, and 6 are considered dissonant. The categorization of intervallic stability allows an expression of how concentrated a piece of music is in terms of intervallic stabilty. Figure 6.1 shows a legend of interval-class consonance and dissonance.

Figure 6.1 – Interval class stability.

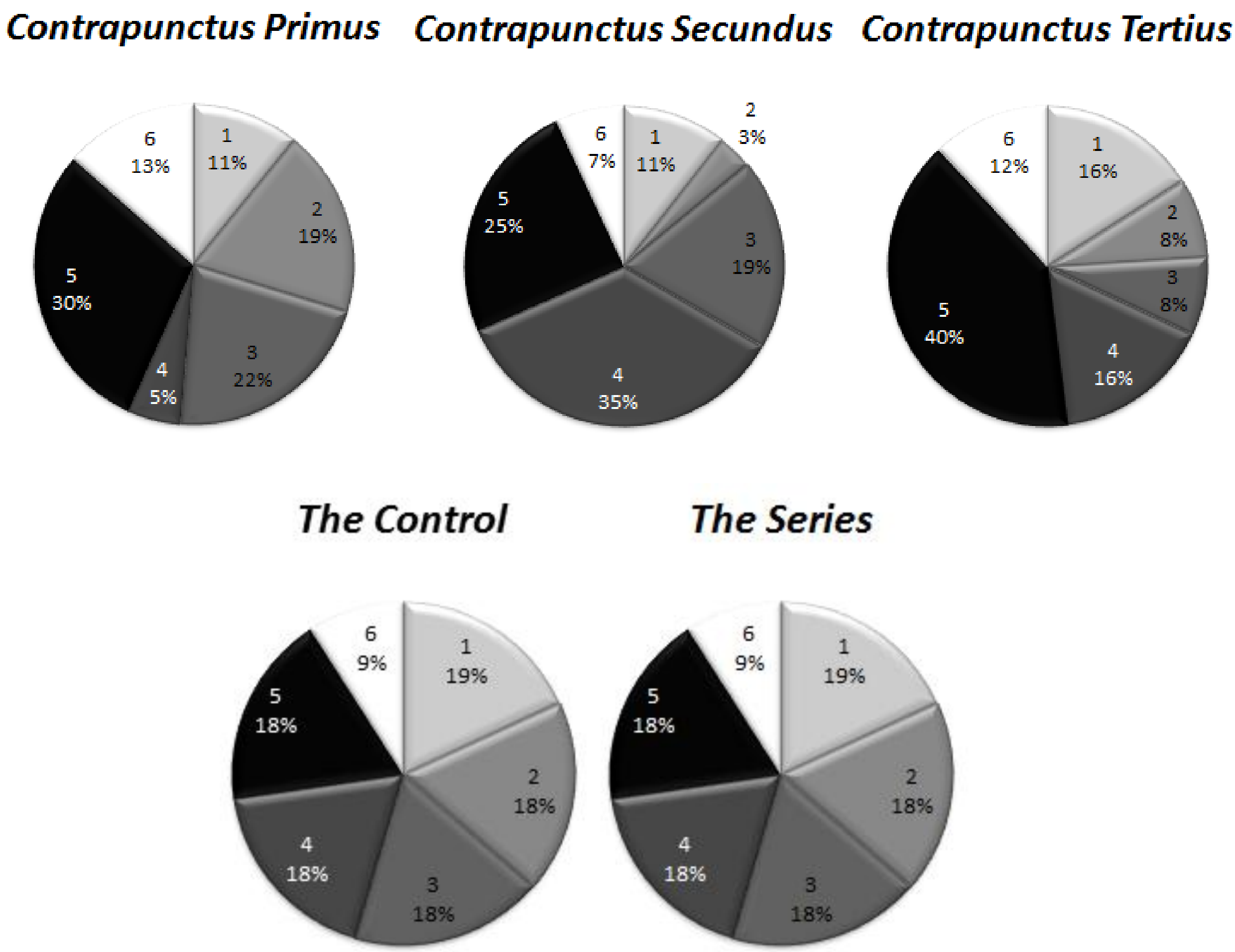
Interval Class	5	4	3	2	1	6
	P5-P4	M3-m6	m3-M6	M2-m7	m2-m7	A4-d5

This scale is not to suggest a hierarchy in the twelve-note system – interval class X is more important than interval class Y – rather, it is only to demonstrate that Dallapiccola frequently used particular interval classes as centric ideas in his music. In chapter three I discussed the topic of polarity as an interval class (referring to *Primus* and *Tertius*), and as a pitch class with an

[20] Luigi Dallapiccola, "On the Twelve-Tone Road", *Music Survey 4* (1951): 322.

axis of symmetry (referring to *Secundus*). I concluded that interval class 5 was the centric musical idea for both *Primus* and *Tertius.* This can be observed in the pie charts (see figure 6.3) – both charts for *Primus* and *Tertius* are more than 50% consonant and have the highest concentration of interval class 5. Although *Secundus* has an inversional axis of symmetry, Dallapiccola takes the necessary melodic and harmonic steps to ensure the saturation of consonant interval classes. The entire twelve-note collection, which I refer to as the *Control*, is approximately 18% for each interval class, with the exception on interval class 6, which is only 9% of the twelve-tone collection. If an interval class concentration is more than 17%, it is considered to play a significant role in the piece and warrants the attention of the listener and the analyst. I have also included the frequency and quality of melodic interval classes in the series. The *Series* representation of data (see figure 6.3), in comparison to any one of the three movements, shows how much effort Dallapiccola took in rearranging the series for the given piece.

Figure 6.3 – Pie chart representation of the data set collected from *Primus, Secundus, Tertius,* the *Control,* and the *Series*.



Dallapiccola believed that notes can take on a new meaning by being arranged in a different way.[21] It is necessary to include the data set for the melodic content of the series because

[21] Dallapiccola, Luigi. *On the Twelve-Tone Road* (London: Farber and Farber, 1951), 324.

Dallapiccola often perceived writing a tone-row as a melody. In Hans Nathan's compilation of *Luigi Dallapiccola: Fragments from Conversations (1966)*, Nathan asks two questions:

> (H.N.: *When you write a row, do you write a melody?*)
> Usually yes. I see to it that the row has a physiognomy of its own. Look at the melodic line in the *Sex Carmina Alcaei* that open the first movement - of this one I have always thought very highly, especially this one...
>
> (H.N.: *Your rows then are conceived in* cantabile *style?*)
> In some works without a doubt. - I maintain that in writing for chorus, for example, as in *Canti Liberazione*, one can insist upon many things, including intonation, provided however that the melodic line has a *cantabile* character, even if it consists of groups of two or three tones.[22]

Although there is no mention in the Nathan interview of the series that was constructed for the *Quaderno*, Dallapiccola's *Canti di Liberazione* was composed during the same time as the *Quaderno* and used the same series. Note that the pie chart representation of the *Control* and the *Series* are identical (see figure 6.3). The data collected for the *Series* was taken from the number of melodic skips between the pitches. One can observe from figure 6.2 that there are two occurrences of each interval class, except for interval class 6 there is only one occurrence. Dallapiccola has created an *all-interval row*[23].

Figure 6.2 - Data collection for the *Series.*

P_0:	**0**		**1**		**5**		**8**		**t**		**4**		**3**		**7**		**9**		**2**		**e**		**6**
Interval Class:		1		4		3		2		6		1		4		2		5		3		5	

[22] Hans Nathan, "Luigi Dallapiccola: Fragments of Conversations" The Music Review Vol. 27, No. 4 (1966): 298.
[23] *All-interval row* - a series whose successive intervals include each interval class twice with two occurrences of interval class 6, once within the series and between the first pitch and the last.

Although, Dallapiccola claims to have constructed the series with the intension of creating an all-interval row[24], his attention to intervallic content gives him all the advantages of working with series that is saturated with every type of interval class.

As previously mentioned in chapter three, a conclusion based entirely on the frequency of an interval class would be considered a superficial one. This is why it is essential that each occurrence recorded falls under the criteria associated with the appropriate piece. Nevertheless, I would like this method of data representation to be only a primary step in determining centric ideas of a piece of serial music. Once there is a reason for further investigation, an analyst can interpret his or her findings by his or her own means. I have included an organized chart of my data collected as well as marked and labeled scores of the interval classes taken from each movement. Table 6.1 is the data set collected for all three movements as well as the *Control* and the *Series.*

Table 6.1 – Data set for *Primus, Secundus, Tertius,* the *Control,* and the *Series.*

Interval Class	***Primus***		***Secundus***		***Tertius***		***The Control***		***The Series***	
1	4	11%	12	11%	12	17%	12	19%	2	19%
2	7	19%	4	3%	6	7%	12	18%	2	18%
3	8	22%	22	19%	6	14%	12	18%	2	18%
4	2	5%	40	35%	12	11%	12	18%	2	18%
5	11	30%	28	25%	30	39%	12	18%	2	18%
6	5	13%	8	7%	9	14%	6	9%	1	9%

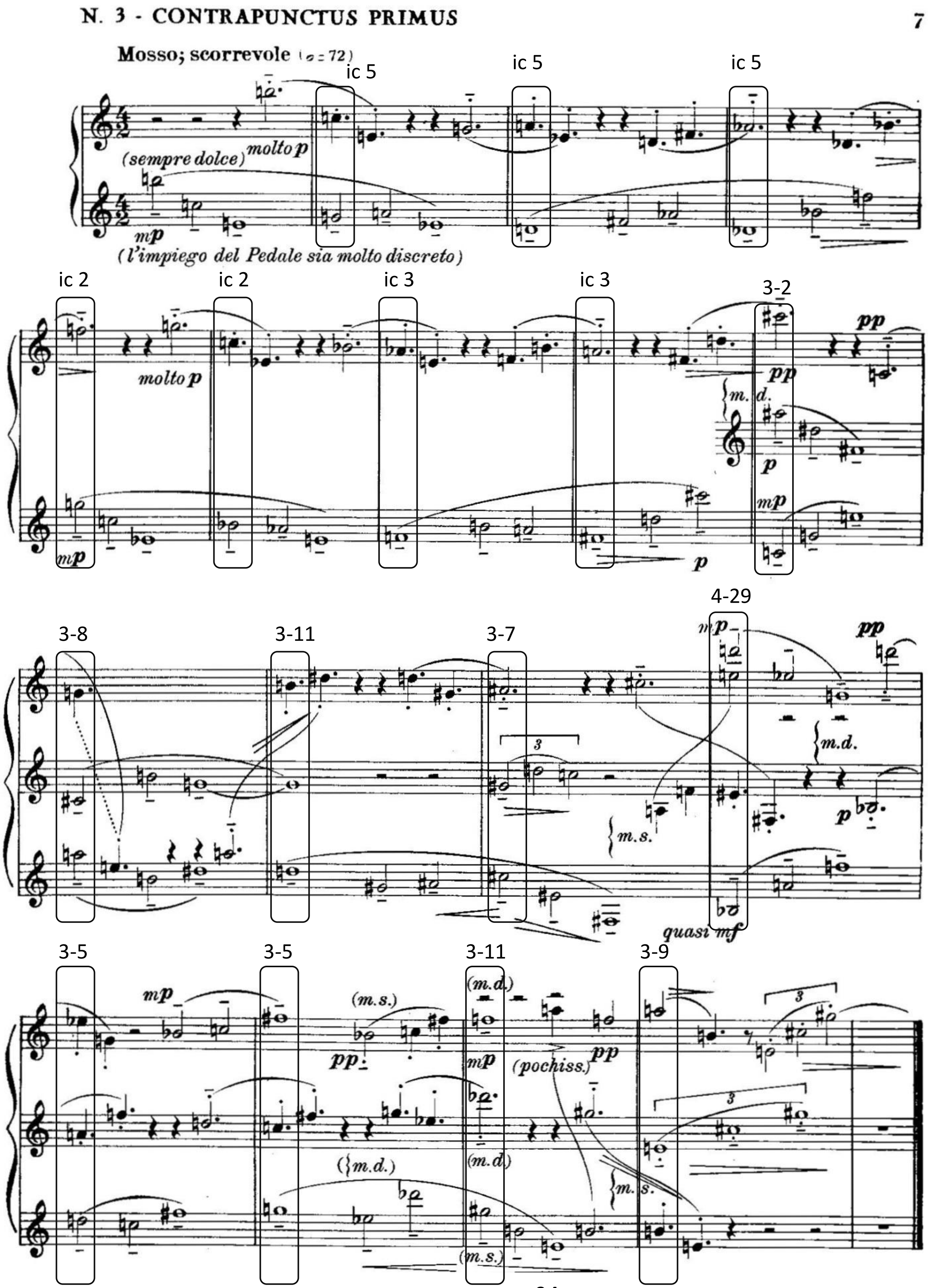
This data is to suggest that Dallapiccola was aware of these recurring relationships and took them into consideration when writing his compositions. The move from tonality to atonality was not so much a choice, but rather the result of musical necessity. Dallapiccola

[24] Luigi Dallapiccola (Translated by F. Chloë Stodt), "Notes for an Analysis of the *Canti di Liberazione*" *Perspectives of New Music* Vol. 38, No. 1 (2000): 7.

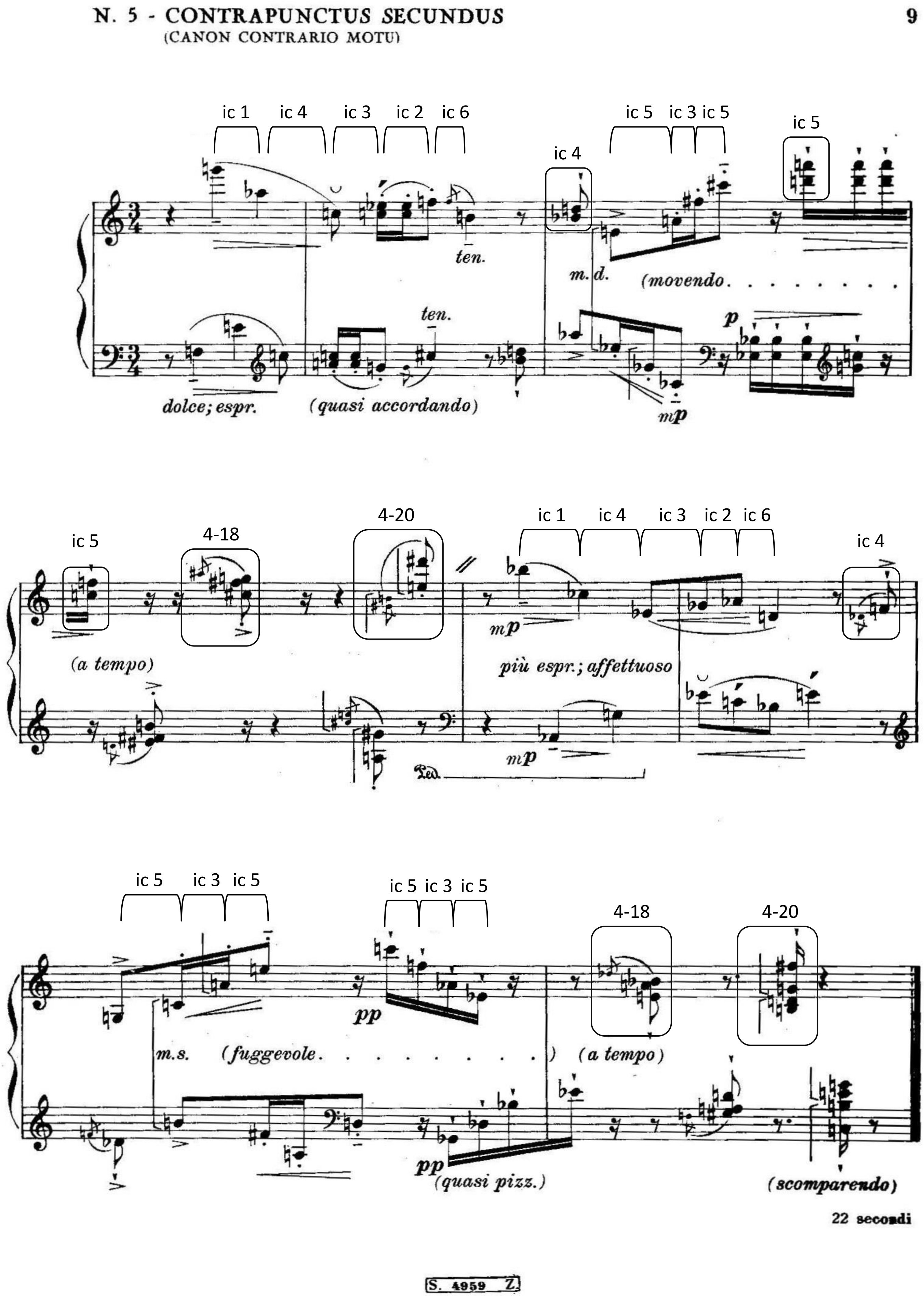
describes this move as something that "allowed us to construct music in a manner comparable to prose rather than to poetry divided by lines - it has shown us a way to expand the length of periods."[25] The twelve-tone system gave Dallapiccola the freedom he was looking for. The restrictions of key signatures did provide enough freedom to achieve optimum expressivity.

[25] Hans Nathan, "Luigi Dallapiccola: Fragments of Conversations" *The Music Review,* Vol. 27, No. 4 (1966): 296.

Example 6.1 – *Contrapunctus Primus*



Example 6.2 – *Contrapunctus Secundus.*



Example 6.3 – *Contrapunctus Tertius.*

AFTERWORD

ARRANGEMENT

Dallapiccola makes use of a single twelve-tone series for all eleven movements of the *Quaderno*. When constructing a series, Dallapiccola would take great care in the arrangement of the pitch classes. Through the close examination of works by the early twentieth-century poet James Joyce, Dallapiccola came to understand that certain musical allusions are comparable to literary passages of Joyce's work, specifically the exploitation of characters. It was Dallapiccola's intention to preserve this literary device, used through words in Joyce's work, in his own compositions. The use of *cancrizans* (commonly referred to as palindromes) was particularly fascinating to Dallapiccola. It was through the admiration of Joyce's work that Dallapiccola had come to realize that the succession of notes can also take on new meaning through a new arrangement.[26] Dallapiccola's view on the arrangement of a series is expressed through the comparison of classical music to the twelve-tone system:

> "In classical music, the theme is nearly always subjected to *melodic* transformation, while its rhythm remains unaltered; in music based on a note-series, the task of transformation is considered with the *arrangement* of the notes, independent of rhythmic considerations."[27]

However, in a system where notes are considered to have equal importance, Dallapiccola considers one element which was not stripped from tonality – time. What he regards as music's *fourth dimension*, the timing of the arrival of notes still holds true in the twelve-tone system. Although there is no tonic-dominant relationship in the twelve-tone system, he argues that

[26] Luigi Dallapiccola, "On the Twelve-Tone Road", *Music Survey 4* (1951): 324.
[27] Ibid., 323.

there is still a power of attraction, what he calls *polarity* (to which we have referred to in our analyses). While such relationships are not so easily noticed as they are in tonal music, they are still existent nonetheless.

Marcel Proust, a French novelist, also drew the attention of Dallapiccola through his subtleties of introducing a character. Dallapiccola writes about one particular character, Albertine, from Proust's *À l'ombre des jeunes filles en fleur*. This character is not formally introduced; rather, Albertine is spoken about occasionally and is attached to qualities that are important to the protagonist and, therefore, forces the reader to pay particular attention to Albertine. Though little is known about Albertine, Proust uses luring traits to engage the reader's interest. It is not until the third book of *À l'ombre des jeunes filles en fleur* and the eighth time her name is mentioned that the reader is actually introduced to her. It is this subtle and enigmatic technique which Dallapiccola so delicately imbeds throughout his own compositions. Techniques which do not reveal the series of a piece of music as a single entity, like cross partitioning, are reflective of Proust's technique of engaging the reader. In *Luigi Dallapiccola: Fragments of Conversations* complied by Hans Nathan, Dallapiccola claims that a "row can be a start but we should not assume that it must be heard as an entity from the beginning of a piece to its end…"[28] He speaks again on harmony: "I have never nourished myself from musical journals in which they explain the chord progressions one should use. No, I wanted to arrive through my own conviction – at the cost of arriving sometimes a little late…"[29] This is, perhaps, the single most descriptive statement that accurately summarizes what is so Italian about Dallapiccola's music. Falling slightly under the Italian stereotype, Italians always

[28] Hans Nathan, "Luigi Dallapiccola: Fragments of Conversations" The Music Review Vol. 27, No. 4 (1966): 299.
[29] Ibid, 300.

make you wait; Dallapiccola's concept of slowly revealing his musical ideas so that the listener's ears are sitting on the edge of their aural seats becomes the most important aspect of his composition.

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